Mathematical Model of Horse and Rider Interaction during Horse Jumping

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***Abstract*—To understand the horse-human interaction, their movement during trot are successfully modeled by spring-damper-mass (SDM) models. However, whether the SDM models are applicable to jump is not clear since jump is not an oscillatory. To examine the applicability of the SDM models, we evaluated the prediction ability of the two SDM models, an SDM with a forcing function (Model 1) and an SDM with an active spring-damper system (Model 2), using the trajectories of the centers of gravity of the horse and the rider during jump collected from videos, which include canter and jump. As a result, Both Models 1 and 2 succeeded to predict the observed trajectories for the canter data, however, only Model 2 succeeded during jump for the jump data. This implies that the rider changes its mechanical property during jump.**

I. Introduction

Horse-riding is the interaction between a horse and its rider and the rider's skill affects the movement of the horse. In fact, the experience of the rider changes the oscillation properties of the horse's movement during trot [1]–[4]. Trot is oscillatory and hence is characterized by the frequency, the amplitude, and the phase, however, jump is not so simple as trot due to its aperiodicity. Thus, jump has been analyzed from various aspects such as the effects of the rider's position [5], the rider's proficiency on jump [6], and the effects of the angular momentum between the horse and the rider [7], [8]. However, how the rider controls the body for jump is still unclear since the rider's movement during jump is not quantified yet. One quantification method is to make a biomechanical model [9]. Biomechanics is a standard approach to analyze human locomotion such as running and walking, where parts of a human are assumed to be rigid [10], [11]. One rigid-body dynamics is spring-damper-mass (SDM) models. SDM models are simple since it regard a mass as a point but they have successfully analyzed human locomotion [12]–[15]. SDM models are also applied to the horse-human interaction during trot [16].

In the horse-human interaction analysis in [16], that is, modeling the vertical displacement of a rider during trot, although the simple spring-mass model failed to explain the displacement, two spring-damper-mass models succeeded it. Here, one of the two models consists of dampers, free fall systems, and a rider's forcing function in addition to the simple model, while the other equips a stiffness-varying spring, which corresponds to the legs of the rider.

In this paper, we examined whether these models also apply to jump. To do it, we collected the trajectory data of a horse and its rider from videos, estimated the parameters of the SDM models, and saw the estimation error of the trajectories. As a result, the second model failed to reproduce the trajectories but the third one succeeded it. This implies the control of the rider's legs is more important during jump than trot.

II. Materials and Methods

A. Data collection and processing

Two videos for jump including canter for the approach run (Data 1, URL: https://www.youtube.com/watch?v=GS8WGSPZAKU fps: 29.7 and Data 2, URL: https://www.youtube.com/watch?v=Gtn2W8-QbjI fps: 25.0) were collected from the Internet videos site (Youtube). From the two videos, anatomical points of the riders and the horses were extracted by using DeepLabCut (a tool for markerless pose estimation of body parts based on deep learning) [17]. The scales (pixel/m) of the videos were calibrated so that the horse withers height was 1.6 m.

The anatomical points for a rider were the head, the shoulders, the hip, the elbows, the wrists, the fingertips, the knees, the ankles, the heels, and the toes. The extracted points with low likelihood for estimation and those that appear wrong were manually corrected. The anatomical points were used to calculate the center of gravity (CoG) of each of the four body parts (the upper body, the upper legs, the lower legs, and the feet) as done in [18] and the CoG of the rider using

$$z_{\mathrm{G}} = \sum_{i=1}^{n} \frac{z_{\mathrm{G},i} m_i}{m}, \quad (1)$$

where m is the mass, $z_{\mathrm{G},i}$ and m_i are the CoG position and the mass in each body parts, respectively. The mass of the rider was set to 60 kg.

The anatomical points for a horse were the eighteen points and were used to calculate the CoG of each of the five body parts (the head, the neck, the trunk, the shoulders, and the thighs) as done in [19] and the CoG of the horse using (1). Note that we used more parts to calculate the CoG of the horse than the previous study [16] where the CoG was the average of the spinous processes of the sixth thoracic and the firt lumbar vertebrae, because the horse moves its neck to maintain the body balance during jump [5]. The mass of the horse was set to 600 kg.

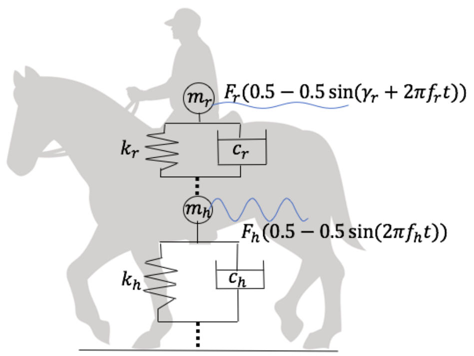


Fig. 1. Model 1. SDM model with forcing function of the rider.

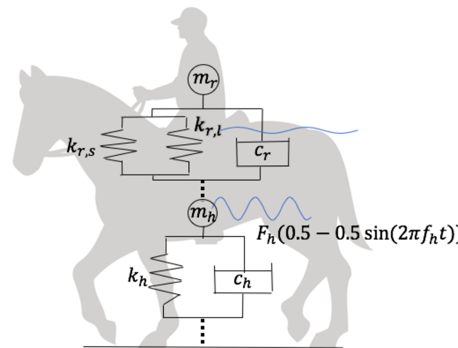


Fig. 2. Model 2. SDM model with an active spring-damper system.

The CoGs were calculated frame by frame and then filtered by Savitzky–Golay filter [20] for smoothing.

B. Spring-damper-mass models

We employed two spring-damper-mass models based on the ones proposed in [16] to see whether the models for trot work during jump.

In the first model (Model 1), the bodies of a horse and a rider are masses m_h and m_r and the legs of the horse and the rider are modeled by springs with stiffness k_h and k_r and dampers with coefficient c_h and c_r (Fig. 1). Model 1 also introduced the contact factor η_r, which represents the contact between the horse and the rider, taking into account that the rider sometimes in a free-fall state apart from the horse. In addition, we assume that the rider moves according to a sinusoidal forcing function with amplitude F_r, phase difference γ_r, and the frequency f_r. In total, the dynamics of Model 1 at time t is described as

$$\begin{aligned} m_r \ddot{z}_r &= -\eta_r c_r(\dot{z}_r - \dot{z}_h) - \eta_r k_r \varepsilon_r \\ &\quad - m_r g + \eta_r F_r(0.5 - 0.5\sin(\gamma_r + 2\pi f_r t)), \quad (2) \end{aligned}$$

$$\varepsilon_r = \frac{(z_r - z_h) - z_{r,\eta}}{z_{r,\eta}}, \quad (3)$$

$$\eta_r = \frac{1}{1 + \exp(a\varepsilon_r)}, \quad (4)$$

where z_h and z_r denote the vertical displacement of the horse and the rider, ε_r is the strain of the rider's legs, η_r is the normalized contact factor, $z_{r,\eta}$ is the difference of the heights of the rider and the horse, g is the constant gravitational acceleration, and a is a constant that determines the maximum value of the contact factors. We chose a so that the maximum takes almost one (0.99 for $a = \log 99$). In this study, $z_{r,\eta}$ was calculated as the average during jump or canter.

In the second model (Model 2), the spring-damper system for the legs of the rider in Model 1 is replaced with an active spring-damper system that has two springs with constant stiffness $k_{r,s}$ and variable stiffness $k_{r,l}$. The stiffness $k_{r,l}$ takes a sinusoidal value from $k_{r,l,base}$ to $k_{r,l,base} + k_{r,l,amp}$ with phase difference γ_r and the frequency f_r, instead of the forcing function in Model 1. In addition, the two springs have the contact factors $\eta_{r,s}$ and $\eta_{r,l}$, respectively. In total, the dynamics of Model 2 at time t is described as

$$\begin{aligned} m_r \ddot{z}_r &= -\eta_{r,c} c_r(\dot{z}_r - \dot{z}_h) \\ &\quad - \eta_{r,s} k_{r,s} \varepsilon_{r,s} - \eta_{r,l} k_{r,l} \varepsilon_{r,l} - m_r g, \quad (5) \\ k_{r,l} &= k_{r,l,base} + k_{r,l,amp}(0.5 - 0.5\sin(\gamma_r + 2\pi f_r t)), \quad (6) \\ z_{r,\eta_l} &= z_{r,\eta_l,base} - z_{r,\eta_l,amp}\sin(\gamma_r + 2\pi f_r t), \quad (7) \end{aligned}$$

$$\varepsilon_{r,s} = \frac{(z_r - z_h) - z_{r,\eta_s}}{z_{r,\eta_s}}, \quad (8)$$

$$\varepsilon_{r,l} = \frac{(z_r - z_h) - z_{r,\eta_l}}{z_{r,\eta_l}}, \quad (9)$$

$$\eta_{r,s} = \frac{1}{1 + \exp(a\varepsilon_{r,s})}, \quad (10)$$

$$\eta_{r,l} = \frac{1}{1 + \exp(a\varepsilon_{r,l})}, \quad (11)$$

$$\eta_{r,c} = \begin{cases} \eta_{r,s} & (\eta_{r,s} \geq \eta_{r,l}), \\ \eta_{r,l} & (\eta_{r,s} < \eta_{r,l}), \end{cases} \quad (12)$$

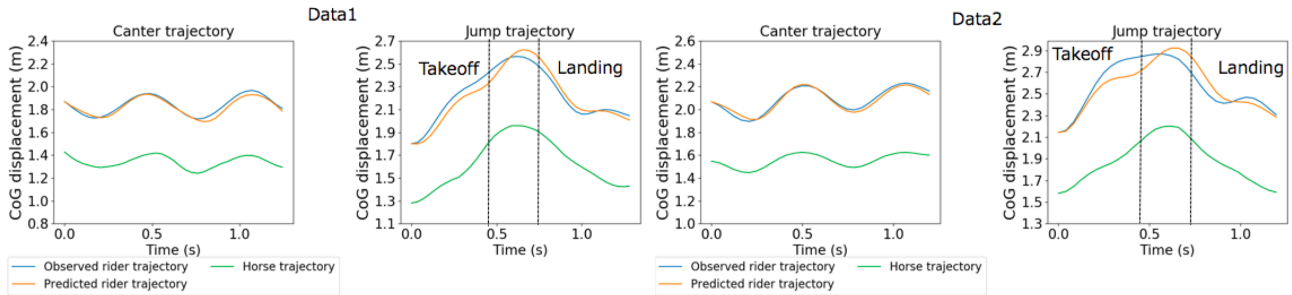
C. Parameter estimation and trajectory prediction

In this study, we estimated the parameters in the models from the observed displacements, z_h and z_r, using Differential Evolution [21], an evolutionary algorithm for optimization. Here, the objective function for the optimization was the mean square error of the rider's CoG displacement, as done in [21]. The search range of the parameters are presented in Table. I, which was determined from the values in [12], [15], [16], [22].

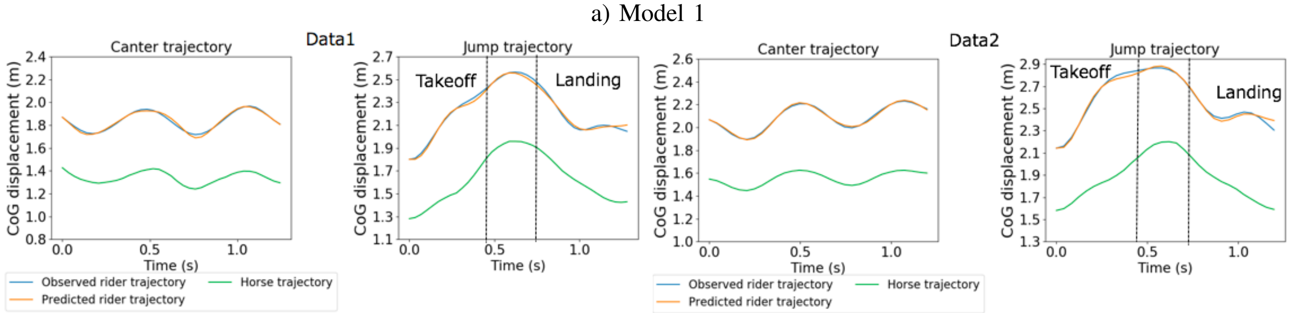
To see how much the models with the estimated parameters explain the observed trajectories of the rider during jump and canter, the differential equations in the models were solved using RK45 in Python scipy liblary solve_ivp. Here, the trajectories of the horse, z_h, were given from the videos.

III. Results

We compared the predicted trajectories from the horse trajectories by Models 1 and 2 with the observed trajectories during jump or canter and found that both models succeeded to predict the observed trajectories during canter but only Model 2 succeeded during jump (Fig. 3). In particular, the trajectories



a) Model 1



b) Model 2

Fig. 3. Observed and predicted trajectories of the CoGs.

by Model 1 are apart from the observed ones after the horse and the rider approach their highest positions during jump.

The difference between the two models is a forcing function and an active spring-damper system. Thus, to see the difference in more details, the net spring stiffness in Model 1, $\eta_r k_r$, and that in Model 2, $\eta_{r,s} k_{r,s}$ and $\eta_{r,l} k_{r,l}$, were calculated. As results, the net spring stiffness in Model 2 drastically changes at takeoff and landing, while that in Model 1 takes almost a constant value (Fig. 4).

IV. Discussion

To understand the horse-human interaction during jump from the biomechanical viewpoint, we examined whether two SDM models based on the models in [16], which succeeded to predict the trajectories during trot, can predict the trajectories during jump, using the collected trajectory data of a horse and its rider from videos. Model 1 is an SDM model with a forcing function and Model 2 is an SDM model with an active spring-damper system, whose mechanical parameters of the models were estimated from the observed trajectories. Both Models 1 and 2 with the estimated parameters succeeded to predict the observed trajectories for the canter data, however, only Model 2 succeeded during jump for the jump data (Fig. 3). The difference may result from the active spring-damper system. In fact, the net spring stiffness in Model 2, $\eta_{r,l} k_{r,l}$, takes a high value before the takeoff, significantly decreased after the takeoff, and increased back after the landing (Fig. 4). In contrast, Model 1 failed to predict the trajectories during jump. This is because Model 1 little changes the net spring stiffness. Although we applied our models to only two videos, their results are consistent.

The decrease of the stiffness reduces the height of the CoG of the rider. This may reduce the power/energy the horse is necessary to produce. In fact, the rider makes the angular momentum against the CoG of the horse minimal so that the rider minimally affects the horse [7], [8]. To confirm this, we need to extend our one-dimensional model to a two-dimensional one and take into account the angular momenta of the rider and the horse.

The rider's movement is important for jump, since the rider's proficiency may also affect the movement of the horse [5]. To examine this effect, we need not only to extend the model to a two-dimensional but also collect more data of riders with different levels of the riding skill.

TABLE I
Search Range of the Model Parameters

	search range
damping coefficient c_r (kgs^{-1})	0-3000
spring stiffness $k_r, k_{r,s}$(kNm^{-1})	0-80
active spring base stiffness $k_{r,l,base}$ (kNm^{-1})	0-40
active spring increase stiffness $k_{r,l,amp}$ (kNm^{-1})	0-40
amplitude of the forcing function F_r (N)	0-1200
phase difference of the forcing function γ_r	0-2π
amplitude of the rider's leg $z_{r,\eta_l,amp}$ (m)	0-0.3
rider's frequency f_r (Hz)	0-3

References

[1] J. Lagarde, C. Peham, T. Licka, and J. S. Kelso, "Coordination dynamics of the horse-rider system," *Journal of motor behavior*, vol. 37, no. 6, pp. 418–424, 2005.

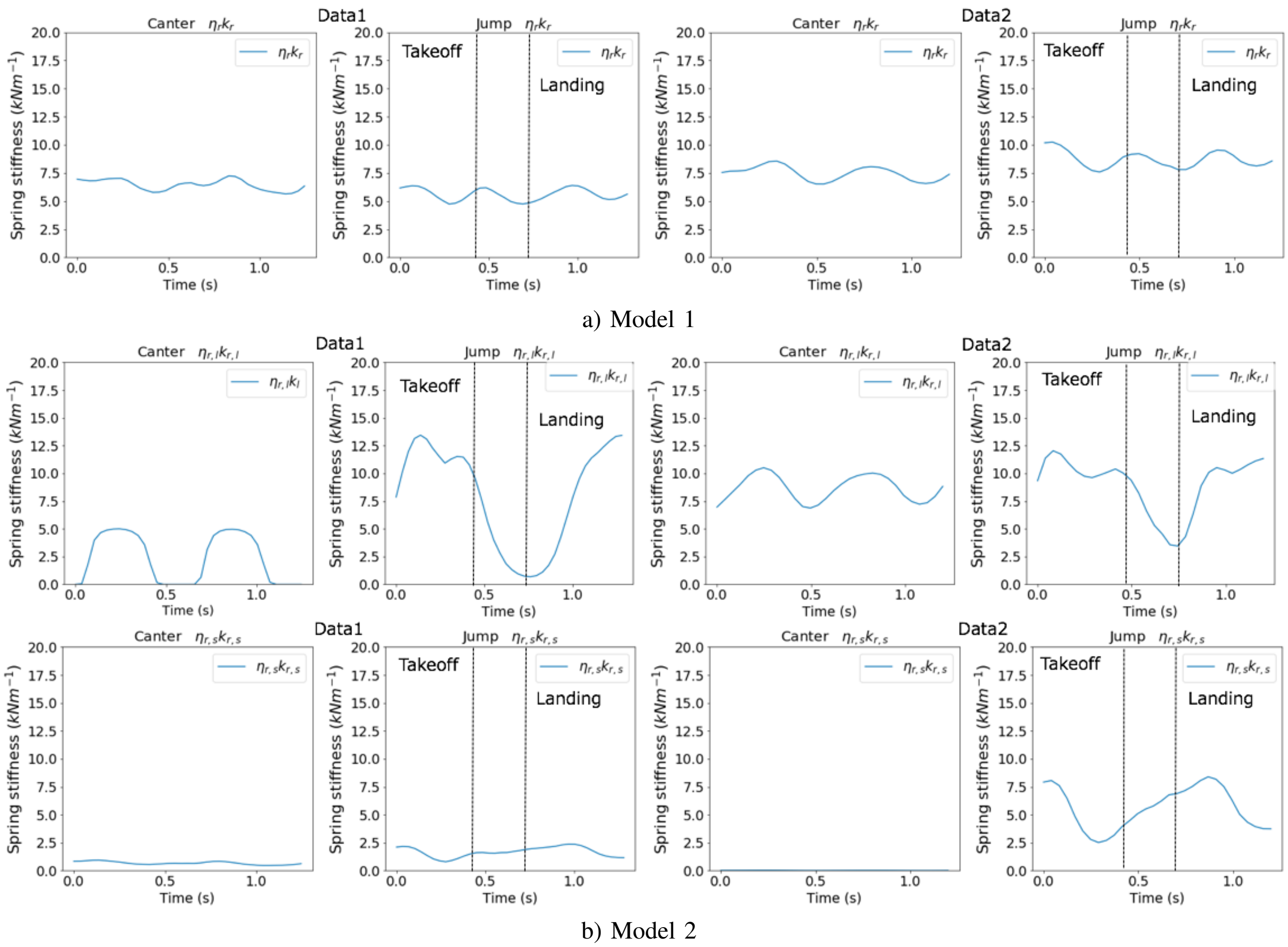


Fig. 4. Net spring stiffness $\eta_r k_r$ of the rider.

[2] S. J. Schils, N. L. Greer, L. J. Stoner, and C. N. Kobluk, "Kinematic analysis of the equestrian—walk, posting trot and sitting trot," *Human movement science*, vol. 12, no. 6, pp. 693–712, 1993.

[3] O. D. Kang, Y. C. Ryu, C. C. Ryew, W. Y. Oh, C. E. Lee, and M. S. Kang, "Comparative analyses of rider position according to skill levels during walk and trot in jeju horse," *Human movement science*, vol. 29, no. 6, pp. 956–963, 2010.

[4] A. Münz, F. Eckardt, and K. Witte, "Horse–rider interaction in dressage riding," *Human movement science*, vol. 33, pp. 227–237, 2014.

[5] P. Nemecek, L. Cabell, and M. Janura, "Horse and rider interaction during simulated horse jumping," *Journal of Equine Veterinary Science*, vol. 70, pp. 26–31, 2018.

[6] P. N. Powers and A. M. Kavanagh, "Effect of rider experience on the jumping kinematics of riding horses," *Equine and Comparative Exercise Physiology*, vol. 2, no. 4, pp. 263–267, 2005.

[7] P. Powers and A. Harrison, "Influences of a rider on the rotation of the horse–rider system during jumping," *Equine and Comparative Exercise Physiology*, vol. 1, no. 1, pp. 33–40, 2004.

[8] P. Galloux and E. Barrey, "Components of the total kinetic moment in jumping horses," *Equine Veterinary Journal*, vol. 29, no. S23, pp. 41–44, 1997.

[9] A. A. Nikooyan and A. A. Zadpoor, "Mass–spring–damper modelling of the human body to study running and hopping–an overview," *Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine*, vol. 225, no. 12, pp. 1121–1135, 2011.

[10] W. Liu and B. M. Nigg, "A mechanical model to determine the influence of masses and mass distribution on the impact force during running," *Journal of biomechanics*, vol. 33, no. 2, pp. 219–224, 2000.

[11] W. Kim, A. S. Voloshin, and S. H. Johnson, "Modeling of heel strike transients during running," *Human Movement Science*, vol. 13, no. 2, pp. 221–244, 1994.

[12] Y. Blum, S. W. Lipfert, and A. Seyfarth, "Effective leg stiffness in running," *Journal of biomechanics*, vol. 42, no. 14, pp. 2400–2405, 2009.

[13] E. Etenzi and V. Monaco, "Bipedal spring-damper-mass model reproduces external mechanical power of human walking," in *2015 37th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*. IEEE, 2015, pp. 2446–2449.

[14] H. Hobara, K. Inoue, K. Gomi, M. Sakamoto, T. Muraoka, S. Iso, and K. Kanosue, "Continuous change in spring-mass characteristics during a 400 m sprint," *Journal of Science and Medicine in Sport*, vol. 13, no. 2, pp. 256–261, 2010.

[15] A. A. Zadpoor and A. A. Nikooyan, "Modeling muscle activity to study the effects of footwear on the impact forces and vibrations of the human body during running," *Journal of biomechanics*, vol. 43, no. 2, pp. 186–193, 2010.

[16] P. de Cocq, M. Muller, H. M. Clayton, and J. L. Van Leeuwen, "Modelling biomechanical requirements of a rider for different horse-riding techniques at trot," *Journal of experimental biology*, vol. 216, no. 10, pp. 1850–1861, 2013.

[17] A. Mathis, P. Mamidanna, K. M. Cury, T. Abe, V. N. Murthy, M. W. Mathis, and M. Bethge, "Deeplabcut: markerless pose estimation of user-defined body parts with deep learning," *Nature neuroscience*, vol. 21, no. 9, p. 1281, 2018.

[18] P. De Leva, "Adjustments to zatsiorsky-seluyanov's segment inertia

parameters," *Journal of biomechanics*, vol. 29, no. 9, pp. 1223–1230, 1996.

[19] H. Buchner, H. Savelberg, H. Schamhardt, and A. Barneveld, "Inertial properties of dutch warmblood horses," *Journal of biomechanics*, vol. 30, no. 6, pp. 653–658, 1997.

[20] A. Savitzky and M. J. Golay, "Smoothing and differentiation of data by simplified least squares procedures." *Analytical chemistry*, vol. 36, no. 8, pp. 1627–1639, 1964.

[21] R. Storn and K. Price, "Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces," *Journal of global optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[22] M. F. Bobbert and L. J. Richard Casius, "Spring-like leg behaviour, musculoskeletal mechanics and control in maximum and submaximum height human hopping," *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 366, no. 1570, pp. 1516–1529, 2011.