Multi-Beam Design Method for a Steerable Parametric Array Loudspeaker

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***Abstract*—The parametric array loudspeaker (PAL) uses an ultrasonic transducer array (UTA) to transmit inaudible ultrasonic frequencies into air, in order for an audible difference frequency to be produced by the parametric acoustic array (PAA). The PAL is advantageous as it possesses a much narrower sound beam as compared to the conventional loudspeaker of the same size. The mixed Gaussian directivity method has been previously proposed in a single-beam design method of the PAL to approximate the Westervelt's directivity. It allows the convolution model to be analytically decomposed into solvable equations. Due to the nonlinear nature of the PAL, linear combinations of single-beam designs do not readily yield a multi-beam design. For this reason, this paper establishes a specific multi-beam design method and validates its effectiveness by 2D nonlinear acoustic field simulations.**

I. Introduction

The PAA describes the nonlinear process in the generation of new frequency components when an ultrasonic wave consisting of two primary frequencies is propagating in a nonlinear medium [1], [2], [3]. The generated new frequency components include the difference, sum, and harmonic frequencies. High frequency components lead to high absorption coefficients. As a result, the primary frequencies, as well as their sum and harmonic frequencies, decay more rapidly than the difference frequency. More importantly, the difference frequency still exhibits a narrow directivity similar to that of the ultrasonic wave. When two primary frequencies are close enough, their difference frequency becomes audible to human beings.

The PAL is an application of the PAA in air [4], [5], [6]. A PAL is commonly made up of a drive circuit and an UTA. The drive circuit carries out modulation of the desired audio on an ultrasonic carrier. The UTA transmits the modulated ultrasonic carrier with large amplitude. The difference frequency between the carrier and sideband frequencies is thereafter generated by the PAA and provides a slightly distorted waveform of the desired audio [7], [8], [9]. The PAL is often considered to be the most efficient directional sound source in a compact size and is found to be very useful in a wide range of audio applications [10], [11], [12].

The UTA usually consists of hundreds of the piezoelectric ceramic transducers (PZTs). When they are grouped into channels and driven by individual amplifiers, phased array techniques can be adopted to implement the steerable PAL [13], [14], [15]. A diagram of the steerable PAL is shown in Fig. 1. Individual complex channel weights are applied to the carrier and sideband frequencies to form their steering angles, such that the steering angle of the difference frequency is correspondingly manipulated.

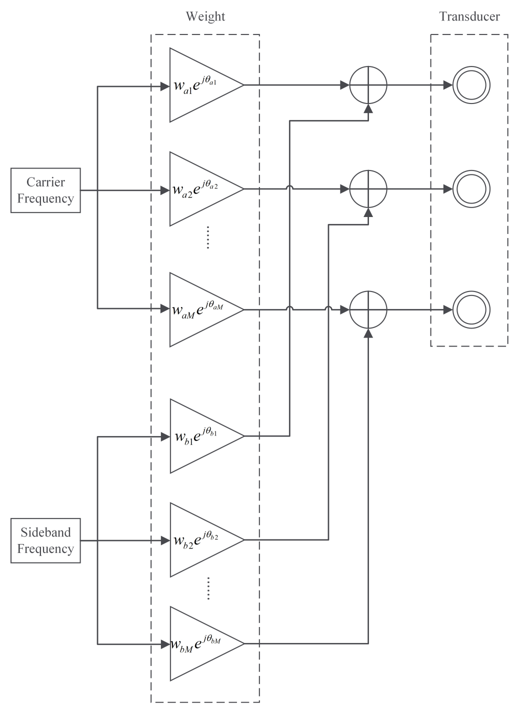

Fig. 1. Diagram of the steerable PAL with complex channel weights (extracted and modified from [16]).

Based on the convolution model, the directivity of the difference frequency can be calculated by the convolution between the product directivity and the Westervelt's directivity, while the product directivity is simply the product of the directivities of the two primary frequencies [16], [17]. Designing the directivity of the steerable PAL is a mathematically underdetermined problem, due to the Westervelt's directivity.

There is an earlier attempt to constrain directivities of the primary frequencies to have sidelobes with equal heights by applying the Chebyshev window weights [18]. The adjustable parameter is limited to the sidelobe level. A lookup table is then prepared for the transmission of a constant-beamwidth directivity of the PAL across a given band of the difference frequency. This method requires a large number of controllable channels in the UTA, which may not be feasible in practice.

Alternatively, a mixed Gaussian directivity method has been proposed in the single-beam design method of the steerable PAL [19]. The Westervelt's directivity is approximated by a linear combination of Gaussian functions. Directivities of the primary frequencies are designed to be Gaussian functions. By doing so, the convolution model can be analytically decomposed into solvable equations. Due to the nonlinear nature of the PAL, linear combinations of single-beam designs do not readily yield a multi-beam design [20]. Therefore, this paper proposes the multi-beam design method based on the mixed Gaussian directivity method and carried out 2D nonlinear acoustic field simulations to validate its effectiveness.

II. Theory

A. Convolution Model

The directivity of the difference frequency $D(\theta)$ can be described by the convolution model, *i.e.*

$$D(\theta) = [D_1(\theta) \times D_2(\theta)] * D_W(\theta), \tag{1}$$

where $D_1(\theta)$ and $D_2(\theta)$ are directivities of the primary frequencies; $*$ denotes the convolution operation; and $D_W(\theta)$ is the Westervelt's directivity, written as

$$D_W(\theta) = \frac{1}{\sqrt{1 + \frac{{\omega_d}^2 \tan^4\theta}{c_0^2(\alpha_1+\alpha_2)^2}}}, \tag{2}$$

where ω_d is the angular frequency of the difference frequency; c_0 is the speed of sound; α_1 and α_2 are the absorption coefficients of the primary frequencies.

B. Mixed Gaussian Directivity

In the mixed Gaussian directivity method, a linear combination of N Gaussian functions provides an approximated substitute of the Westervelt's directivity, *i.e.*

$$D_W(\theta) \simeq \sum_{n=1}^{N} a_n e^{-\left(\frac{\theta-b_n}{c_n}\right)^2}, \tag{3}$$

where a_n, b_n, and c_n are frequency dependent. They can be treated as functions of the difference frequency.

The multiplication of two Gaussian functions is readily given by another Gaussian function, while the convolution between two Gaussian functions can be analytically expressed as well. Therefore, directivities of the primary frequencies are designed as mixed Gaussian functions, in order for the convolution model to be analytically decomposed into solvable equations.

C. Multi-Beam Design Method

Without loss of generality, it is assumed that $D_1(\theta)$ is the directivity of the carrier frequency and $D_2(\theta)$ is the directivity of the sideband frequency. $D_1(\theta)$ is necessary to have a sufficiently large beamwidth, because the beamwidth of the product directivity is narrower than the beamwidth of either $D_1(\theta)$ or $D_2(\theta)$. The directivity of the carrier frequency is firstly fixed as a Gaussian function, which is given by

$$D_1(\theta) = e^{-\left(\frac{\theta}{\beta_1}\right)^2}, \tag{4}$$

where β_1 is an adjustable parameter associated with the half-power beamwidth (HPBW) by a constant factor of $\sqrt{\ln 4}$.

The product directivity is designed as a linear combination of the second order Gaussian functions, which is written as

$$D_1(\theta) \times D_2(\theta) = \sum_{i=1}^{M} a_{p_i} e^{-\left(\frac{\theta-b_{p_i}}{c_{p_i}}\right)^2} \tag{5}$$

where M is the number of beams; a_{p_i}, b_{p_i} and c_{p_i} represent the normalized amplitude, steering angle and HPBW of the i-th beam in the product directivity, respectively. Note that $a_{p_1} = 1$ is ensured by normalization. a_{p_i}, b_{p_i}, and c_{p_i} are frequency dependent. Substituting (2) and (5) into (1) yields

$$D(\theta) = \sum_{i=1}^{M}\sum_{n=1}^{N} \frac{\sqrt{\pi} a_{p_i} a_n c_{p_i} c_n}{\sqrt{{c_{p_i}}^2 + {c_n}^2}} e^{-\frac{(\theta-b_{p_i}-b_n)^2}{{c_{p_i}}^2+{c_n}^2}}. \tag{6}$$

Denote the normalized amplitude, steering angle and HPBW of the i-th beam in the desired directivity as a_{d_i}, b_{d_i} and c_{d_i}, respectively. Note that $a_{d_1} = 1$ is ensured by normalization. There are $3M-1$ equations derived from (6), which are

$$D\left(b_{d_i} \pm \frac{c_{d_i}}{2}\right) = \frac{\sqrt{2}}{2} D(b_{d_i}) \tag{7}$$

and

$$D(b_{d_i}) = a_{d_i} D(b_{d_1}), \tag{8}$$

for $i = 1, 2, \ldots, M$. Hence, a_{p_i}, b_{p_i} and c_{p_i} can be uniquely determined for each difference frequency in a given frequency band. At last, $D_2(\theta)$ is designed as

$$D_2(\theta) = \frac{D_1(\theta) \times D_2(\theta)}{D_1(\theta)} = \frac{\sum\limits_{i=1}^{M} a_{p_i} e^{-\left(\frac{\theta-b_{p_i}}{c_{p_i}}\right)^2}}{e^{-\left(\frac{\theta}{\beta_1}\right)^2}}. \tag{9}$$

D. Calculation of Weights

Calculating the weights of all the channels is based on a given directivity of either the carrier frequency or the sideband frequency. The directivity of a linear array is written as

$$f(\theta) = \sum_{m=0}^{M-1} w_m e^{j\theta_m} e^{\frac{j2\pi m d \sin\theta}{\lambda}}, \tag{10}$$

where $w_m e^{j\theta_m}$ is the complex weight of the m-th channel; d is the unified spacing between two neighbouring channels; and λ is the wavelength. By introducing the normalized variable ψ, (10) is transformed into a frequency response form, *i.e.*

$$f(\theta) = F(\psi) = \sum_{m=0}^{M-1} w_m e^{j\theta_m} e^{jm\psi}, \tag{11}$$

where

$$\psi = \frac{2\pi d \sin\theta}{\lambda}. \tag{12}$$

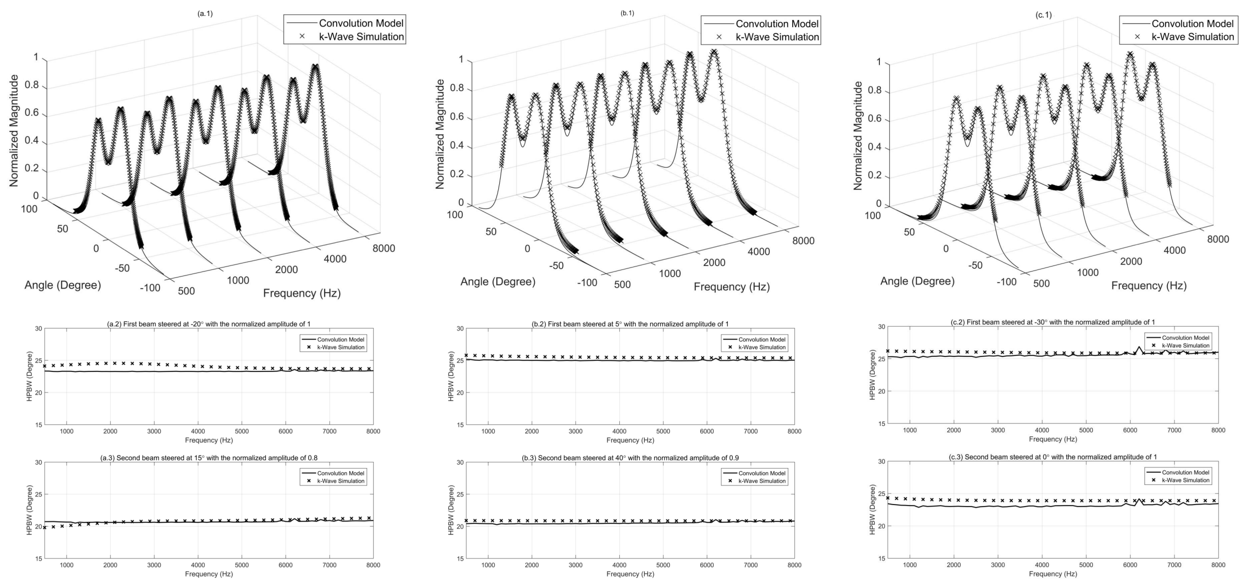

Fig. 2. Double-beam design of the steerable PAL.

When $d \leqslant \frac{\lambda}{2}$, the digital filter design method based on frequency sampling can be readily adopted and the filter coefficients of the digital filter are equal to the complex channel weights of the UTA.

III. SIMULATION AND VALIDATION

In this section, two multi-beam design cases for the steerable PAL are shown with their 2D nonlinear acoustic field simulation results. The 2D nonlinear acoustic field simulations are carried out in the k-wave toolbox [21], [22]. The simulation area is about 2 meters by 6 meters. The UTA is placed on the left and the observation points are located on the right. They are separated by 2 meters in distance. The maximum observation angle achieved in this simulation is $56.3°$ off the axis. The carrier frequency is set to 40 kHz. The beamwidth of the carrier frequency is set to $90°$. This is sufficiently wide as compared to most real-world ultrasonic transducers. The sideband frequency is set to be lower than the carrier frequency, in order for the difference frequency to be generated from 500 Hz to 8000 Hz.

A. Double-beam Design Case

The double-beam design case is illustrated in Fig. 2. The HPBWs extracted from the 2D nonlinear acoustic field simulation are compared with the convolution model results. Five difference frequencies are selected to showcase their directivities. They are 500 Hz, 1000 Hz, 2000 Hz, 4000 Hz and 8000 Hz.

In Fig. 2(a), two beams of the difference frequency are formed at $b_{d_1} = -20°$ and $b_{d_2} = 15°$ with HPBWs of $c_{d_1} = 23°$ and $c_{d_2} = 20°$, respectively. Their normalized amplitudes are given by $a_{d_1} = 1$ and $a_{d_2} = 0.8$, respectively. In Fig. 2(b), beams are shifted to $b_{d_1} = 5°$ and $b_{d_2} = 40°$ with HPBWs of $c_{d_1} = 25°$ and $c_{d_2} = 20°$, respectively. Their normalized amplitudes are adjusted to $a_{d_1} = 1$ and $a_{d_2} = 0.9$, respectively. In Fig. 2(c), the two beams are formed at $b_{d_1} = -30°$ and $b_{d_2} = 0°$ with HPBWs of $c_{d_1} = 25°$ and $c_{d_2} = 23°$, respectively. The normalized amplitudes of the two beams are set equally in this case.

It is validated in Fig. 2 that the proposed multi-beam design method can facilitate the steerable PAL to transmit two beams of the difference frequency at arbitrary angles with different or same HPBWs and normalized amplitudes. The multi-beam design method proposed in this paper is more versatile than methods making use of the spatial aliasing of the primary frequencies [23], [24].

B. Triple-Beam Design Case

Figure 3 illustrates the triple-beam design case. The HPBWs extracted from the 2D nonlinear acoustic field simulation are compared with the convolution model results.

In Fig. 3(a), three beams of the difference frequency are formed at $b_{d_1} = 0°$, $b_{d_2} = -38°$ and $b_{d_3} = 38°$ with HPBWs of $c_{d_1} = 20°$, $c_{d_2} = 25°$ and $c_{d_3} = 25°$, respectively. Their normalized amplitudes are given by $a_{d_1} = 1$, $a_{d_2} = 0.8$ and $a_{d_3} = 0.8$, respectively. In Fig. 3(b), angular separations between beams are narrowed. The three beams of the difference frequency are shifted to $b_{d_1} = -35°$, $b_{d_2} = 0°$ and $b_{d_3} = 35°$ with equal HPBWs of $c_{d_1} = 20°$, $c_{d_2} = 20°$ and $c_{d_3} = 20°$, respectively. Their normalized amplitudes are set equally in this case, i.e. $a_{d_1} = 1$, $a_{d_2} = 1$ and $a_{d_3} = 1$. In Fig. 3(c), the three beams are formed asymmetrically at $b_{d_1} = -37°$,

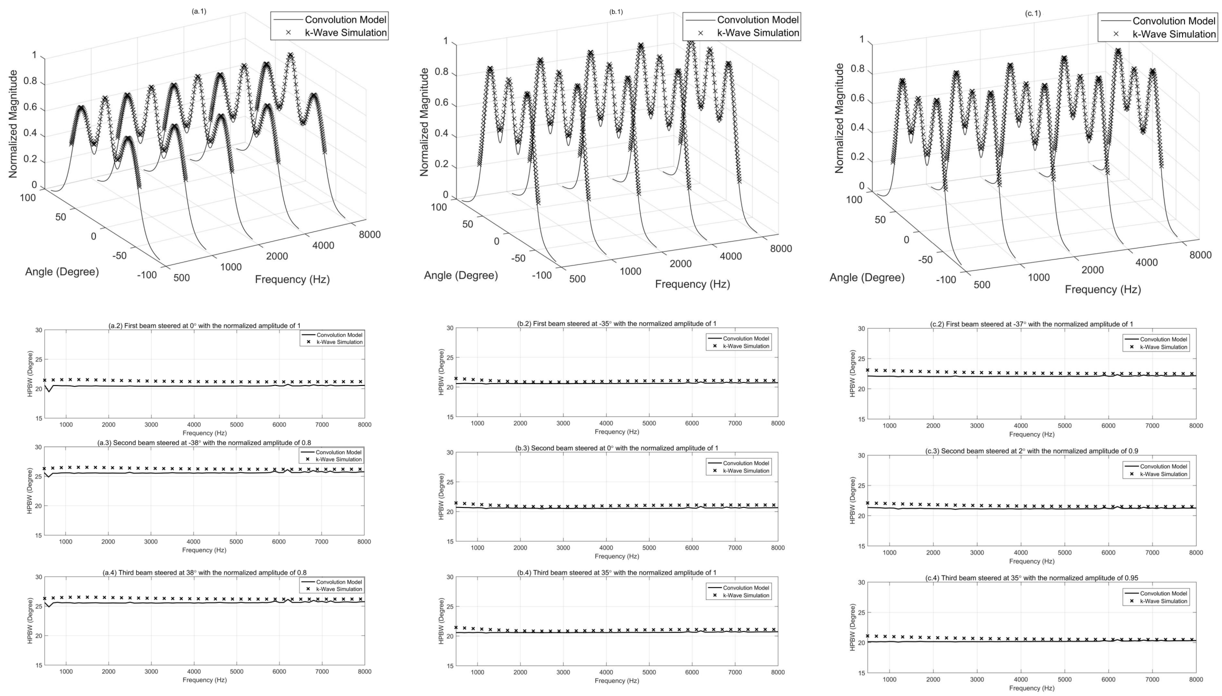

Fig. 3. Triple-beam design of the steerable PAL.

$b_{d_2} = 2^\circ$ and $b_{d_3} = 35^\circ$ with different HPBWs of $c_{d_1} = 22^\circ$, $c_{d_2} = 21^\circ$ and $c_{d_3} = 20^\circ$, respectively. Their normalized amplitudes are given by$a_{d_1} = 1$, $a_{d_2} = 0.9$ and $a_{d_3} = 0.95$, respectively.

In both Figs. 2 and 3, small discrepancies are observed between the convolution model results and the k-wave simulation results. Those discrepancies can be reduced by using more Gaussian functions in the approximation of the Westervelt's directivity. Currently, $N = 6$ is adopted. The numerical error of the 2D nonlinear acoustic field simulation also becomes lower when the difference frequency is higher, as the difference frequency component is easier to be distinguished from the whole spectrum [25]. A similar trend has also been reported in the single-beam design method based on the mixed Gaussian directivity method [19].

IV. Conclusions

This paper proposes a systemic multi-beam design method for the steerable PAL based on the convolution model. The directivity of the carrier frequency is designed as a Gaussian function. The product directivity and the Westervelt's directivity are both considered to be a mixture of several Gaussian functions. The convolution model is thereafter simplified into analytically solvable equations once the number, normalized amplitudes, steering angles and HPBWs of beams are specified. 2D nonlinear acoustic field simulations are carried out in the k-wave toolbox. The simulation results validate the effectiveness of the proposed multi-beam design method when two or three beams are transmitted from the steerable PAL. The discrepancies between the convolution model results and the k-wave simulation results are generally small enough to be negligible in practical implementations.

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