SYMMETRY IN THE STRUCTURE OF MUSICAL NODES

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Abstract-We investigate, if any symmetry that lies ubiquitously in most of the structures of nature is also present in music and how it presents itself within music. We try to quantify symmetry and patterns by constructing the statistical and visual directed graphs of pitch frequencies and their temporal alignment (time duration) in a composition. We draw these graphs for 1,409 tracks. This paper realizes the structure inbetween the pitch and time duration sequences via the underlying probability distributions and graph theory. It reduces the analysis of pitch frequencies in a given composition to 20 % out of 128 and the time durations of in the composition to only 53 out of the range of 0 to 2400 (approximately). Furthermore, we observe symmetric patterns for temporal graphs and their degree distribution indicates a self-organizing behaviour. To model pitch prediction in our brain while listening to a musical composition via Upper Confidence Bound Reinforcement Learning (UCB-RL) algorithm and analyze the learning curves per the composition of the Beethoven, Mozart, and Bach. We observe the highest smallest learning curve for Beethoven and the largest for Bach. This work may find its potential applications in music therapy, music synthesis, and cognitive science.

I. INTRODUCTION

Every sequence of sounds is not music, such as speech, chirping of birds, sounds of moving objects, etc. This implies that a certain structure is present in music which allows us to differentiate it from the rest. Music, when analyzed via a spectrum, shows its harmonics within the frequencies whenever a pitch is played, this can also be visualized as the whirlpool effect seen in the water when a stone is thrown in it. This observation is not unique to music, within speech all the vowel sounds create a harmonic effect, which becomes the basis for many words structured on the vowel sounds [32]. Similarly, within music, everything is constructed within these harmonics [22]. About 70,000 years ago, the first musical instrument was made and used [5]. In today's formalism and language of music, this basis can be identified as the twelve major pitch frequencies which are A, A#, B, C, C#, D, D#, E, F, F#, G, and G#, where _# denote sharp, going into higher or lower octaves gives us their harmonics. The duration for which any pitch is played in the song sequence also plays an important role, which we analyze closely in this paper.

We understand symmetries most easily in terms of visual representations, however, a formulation that allows us to view them in sound is what we investigate in this paper with the help of directed graphs [12, 27]. Coming from the time of music sheet to MIDI scores on a computer, there have been approaches towards the task of automatically retrieving information from music [25]. Matrix factorization methods have been widely used in music for music information retrieval (MIR). A self-similarity matrix representation highlights commonly repeated note sequences which may form the chorus [15]. An initial view of complex networks on music consisted of a collective network of all the compositions of a composer in one graph which concluded the characteristics of music network having a scale-free degree distribution [41]. It also presents a random walk model for music synthesis, based on the network so formed [38]. In the past, similarity (symmetry) has been shown in a scapeplot which consists of notes plotted on a scapeplot sequentially [26, 17, 28]. We observe symmetry within halves of the song, however, we are not able to draw out a cumulative interaction model amongst all the notes and time durations within the scapeplot. Furthermore, the scapeplot forms itself along the time-axis and hence, fails to observe time durations cumulatively, and it is statistically different from the notes.

The study of how our brain reacts to music has been shown broadly in many aspects. By studying the electrical waves emitted by our brain via Electroencephalography (EEG) while listening to good music that we know, creates a pleasure [19]. On the contrary, listening to a sequence of randomly played sounds on the musical instruments leads us to a state of confusion or even displeasure or irritation [18, 14, 21]. This gives the music a position where it has been studied for well being of human beings, for reducing our stress levels, and facilitate the work of neurotransmitters in our body. These play an important role in learning, cognition, sleep behaviour, and memory [33, 20, 29]. It has also been shown to improve our immune system [3]. One of the most important neurotransmitters is dopamine, is released in our brain when we listen to music [21, 34, 8, 35]. The approaches also show that there is an expectationreward system in our brain which leads us to feel good or bad [11], this is prevalent while listening to music. It has been found that it provides a feedback mechanism in our brain via providing a prediction error [4]. Furthermore, investigations show that dopamine influences our internal clock [36]. Models for our dopamine-based distributional reinforcement learning have been used to study the outcomes that our brain predicts [11]. When we can form structures of music, this can be

exploited for understanding which patterns of music lead to what results in the brain, and thus, this knowledge can be used in music therapy for patients.

II. EXPERIMENTAL SETUP

Musical perception is a field that has demanded answers to the long standing questions of music's existence and penetration in our world. To understand music at the holistic-level we break our experimentation into two parts:

- Construct two directed graphs of a single musical composition; the pitch graph and the time durations graph consisting of the interconnections between the pitch frequencies and time durations. respectively.
- Model the musical expectancy behaviour on pitch prediction via an UCB reinforcement learning agent in prediction of the next pitch in a musical composition.

In this study, we intend to decipher the basis of music without the inclusion of linguistic content. Hence, this study consists of only instrumental classical music.

A. Constructing Directed Graphs

The preceptory senses of the human body allow human beings to react to sound stimuli the fastest. However, the visual senses allow us to observe and later quantify the meaning of patterns observed. Hence, through the construction of directed graphs, we intend to construct a visual structure and a statistical structure to extract patterns and key information focusing on a single musical composition. The dataset used for the analysis consists of the MIDI files of three classical music composers, namely, L. V. Beethoven, W. A. Mozart, and J. S. Bach with 119, 297, and 933 tracks, respectively. Please note that in a single composition there are multiple tracks depending ,i.e., a track for each instrument. We first extract the pitch sequences and the time duration sequences from the MIDI files of the compositions [40].

To that effect, let A be the adjacency matrix given by (1):

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix},$$
(1)

with which we construct the adjacency matrix for the two networks both initialized to zero. If $note_i$ is played just before $note_{i+1}$ then, we have (2),

$$a_{note_i,note_{i+1}} = a_{note_i,note_{i+1}} + 1.$$

Similarly, if td_i is just before td_{i+1} then we have (3) :

$$a_{td_i,td_{i+1}} = a_{td_i,td_{i+1}} + 1, (3)$$

where td indicates the time duration. Furthermore, the representation of the graph is constructed following a few guidelines which are:

• The graphs are constructed for a single composition at a time,

- The graph so formed leaves some nodes isolated, which are removed from the graph,
- Each node has a unique colour of edges originating from that node to distinguish its edges in the directed graph so formed, and
- The width of the edges represents the number of times that two nodes are repeated one after the other.

For the pitch graphs as they consist of an innate harmonic structure, we can also visualize the network with the following representations to elicit patterns such as:

- The graph of the harmonics of the same pitch
- A nodewise graph showing a single node edges to other notes.
- A twelve-note graph showing only edges of a single octave at a time, to analyze how they connect to the rest of the nodes, and
- A cumulative graph as can be seen in Fig. 1 and other statistically relevant combinations.

This model can be tweaked to show the edges of any combination of nodes, wherein we only have to specify which nodes do we want to analyze the network of, out of which a few are specified above. The network shown here consists of the edges of all the notes. We do not change how many notes are shown in the graph, however, only control which set of notes do we show the connections of. However, the set of notes shown in the graph depends on the usage of the notes in the composition, which may vary from one composition to the other. The graph's adjacency matrix can be used to draw out probabilities. The probability of the next pitch being $pitch_i$ after a particular pitch pitch can be estimated in (4):

$$P(pitch_i/pitch) = \frac{a_{pitch,pitch_i}}{\sum_{j=1}^{n} a_{pitch,pitch_j}}.$$
(4)

Similarly, in (5) for the next duration being td_i from a particular td can be calculated as where td stands for time duration, i.e., :

$$P(td_i/td) = \frac{a_{td,td_i}}{\sum_{j=1}^{n} a_{td,td_j}}.$$
 (5)

Furthermore, the Probability Distribution Function (PDF) can be drawn out for a single note consisting of the probabilities for all the next notes that can be played. This crucial property allows us to formalize the patterns in the song via Bayes theorem [13]. We can further evaluate the probability of three width sequences, four width sequences, and so on. This would allow us to analyze n length sequences. Evaluating which one has the maximum probability of occurrence embed the sequential properties of music in a composition.

B. Modeling Pitch Perception via Expectancy

The most common process while humming along to a song is predicting the next pitch or the next time duration. As the granularity of the time duration is very high, we only model the prediction of the pitch. To model the predicting behaviour, we use a reinforcement learning algorithm because it models the dopamine learning behaviour that occurs in our brain. To implement reinforcement learning, we compare two algorithms for the task of pitch prediction, namely, Epsilon-Greedy (ϵ – *Greedy*) and Upper Confidence Bound (UCB). Comparing the results of the two, we gain a better prediction for the UCB algorithm and hence, use that for our further analysis. We perform pitch prediction on the following compositions given in Table I.

 Table I

 COMPOSITION SELECTED FOR EACH COMPOSER

Name of Composer	Name of Composition
L. V. Beethoven	Rage over lost penny
W. A. Mozart	Piano Concerto No.5 in D Allegro
J. S. Bach	Inventio 1

III. EXPERIMENTAL RESULTS

A. Construction of Graphs

Reduction and visually symmetric patterns are observed during the construction of the pitch graphs and the time duration graphs. Specifically for reduction, we observe what percent of pitch frequencies (a total of 128) are used in the pitch graph. Similarly, for time duration graphs, we analyze how many nodes form the network out of the wide range of duration present (from 0 to 2400). In Table III-A, we observe the reduction across the pitch frequencies and time durations observed for the tracks of Beethoven, Mozart, and Bach. This indicates that despite the wide range of possibilities, a reduction in the number of pitch frequencies and time durations allow the relatively fast processing of the data constituting the musical composition.

Table II RESULTS ON THE REDUCTION FOR ANALYSIS OF NETWORKS

	Beethoven	Mozart	Bach
# Tracks	119	297	993
Avg. # of notes (%)	20.50	21.70	18.80
Avg. # of time durations	87.44	40.99	32.36

In Table III-A indicates that only about 20 % of all the pitch frequencies are used in the pitch graph ,i.e., 26 nodes. Furthermore, the graph of temporal nodes (time durations) indicates that the number of nodes in the graph is on an average of 53, which is very small compared to the total range of time durations used (ranging from 0 to 2400 approximately). W. A. Mozart's K542 Piano and Winds' music graph shown (Fig.1) highlights broad edges from G#4 to G#5, G#5 to G#6, C4 to C5, and C5 to C6, indicating that these sequences are repeated many times throughout the composition. In addition, this composition consists of the usage of only 58 pitch frequencies out of the 128 present. This representation can be used to identify motifs in a musical sequence. Which can be verified directly via the Eq. 4

The music sheet consists of pitch frequencies, and their time durations. For the analysis of interconnections between the time duration, the graph is drawn for ten time durations at once



Figure 1. Pitch graph for W. A. Mozart's K542 Piano and Winds.

(as a greater number of nodes exist for the temporal graph than the pitch). Furthermore, the relative position of nodes indicates their relative duration. For example, nodes placed next to each other have a similar time duration, as compared to the nodes which are placed at a farther distance. The nodes in the graph depend on the number of distinct time duration used in the network for different compositions. Which is why we see variability in the number of nodes in Fig. 2. Furthermore, they are arranged to start from the rightmost point of the circle, and arranged in a clockwise manner from that point in ascending order. The symmetry in the temporal graph as is highlighted by the oval in the diagrams.

The two consecutive pitch frequencies played consist of a perceivable change in time durations used (one played longer than the other). We quantify the different combinations of time durations used by counting the peaks of the triangles and add one more for the base. This may vary depending on the relative closeness of the peaks.

In Fig. 2, we observe symmetrical patterns in the compositions of Beethoven, Mozart, and Bach. For L. V. Beethoven's composition, we observe a triangular repeating pattern, which is perfectly symmetric to the pattern in the second temporal graph instance. This indicates that while composing, he has used a simple pattern of time durations easier to grasp during the course of the musical composition. In W.A. Mozart's composition, we observe a greater number of triangles. This indicates that he has used sequentially different time duration which may be 5 to 6 in total. Similarly, we observe the patterns in J.S Bach's composition, consisting of 2 triangles indicating about 3 different time durations used. We also noted that sometimes we observe bands, this indicates that two different time durations as bands on each side of the band have a relatively close time duration. Overall, they indicate that to



Figure 2. Temporal Graphs: (a,b) L. V. Beethoven's Moonlight Sonata, (c,d) W. A. Mozart's Cosi fan Tutte, and (e,f) J. S. Bach's Inventio 1.

create a contrast, the next time duration is chosen far-off in the circle to create a sense of perceivable contrast for the listener at each note played.

The symmetry observed in the granularity in time proves to be crucial to our perception of music as they form a clear pattern for the prediction of the next time duration. Some studies have speculated that time durations of pitch frequency influence maximally our perception of rhythm [7]. In Fig. 3, we plot the degree distribution of the time networks by plotting the frequency on the Y-axis, and degrees on the X-axis. These degree distributions indicate a power-law distribution in time networks (also called scale-free networks) [2]. The degree distribution of the network is a power-law indicates that a very small fraction of time nodes has an extremely high degree, and a very large fraction of nodes has a small degree. In Fig. 3 we observe a perfect rectangular hyperbola for Beethoven. A mildly distorted one for Mozart, and highly distorted one for Bach. Such networks indicate the self-organizing behaviour [6]. Self-organizing behaviour occurs because, in design, the incoming node connects with a higher probability to the nodes which are already well connected in the graph rather than to those nodes which are sparsely-connected. This shows us how the interconnects of the network influences its behaviour [1]. Which means that though the network may not be organized

by design, it consists of nodes which are well connected to the rest. Creating an organized structure on its own. Our world also follows this behaviour in many aspects.



Figure 3. Degree Distribution: (a) L. V. Beethoven's Moonlight Sonata, (b) W. A. Mozart's Cosi fan Tutte, and (c) J. S. Bach's Inventio 1.

B. Pitch Prediction

In Fig. III-B, we analyze the learning curves of the compositions in Table I, where the Y-axis indicates the accuracy attained, and X-axis the number of epochs. The hyperparameters used in the algorithm are c = 2, $R_{Correct} = +9$, and $R_{Incorrect} = -2$. In order to observe how fast the accuracy reaches up about 25% we limit our observation to 500 epochs. For Beethoven's composition (as given in Fig. III-B (a))we observe that the learning curve reaches 30% within 10 epochs indicating that Beethoven's composition is easier to learn. In Mozart's composition (as given in Fig. III-B (b)), we observe that the accuracy oscillates between 20-21% for 500 epochs, indicating a complex interconnections between the pitch-frequencies that are harder to learn. In Bach's composition (as given in Fig. III-B (c)), we observe that the accuracy oscillates between 17.5% for 500 epochs depicting the steepest learning curve amongst all the three compositions. This learning model allows us to model the learning of the composition amongst its audiences.

IV. SUMMARY AND CONCLUSIONS

In this study, we proposed the formation of visual and statistical directed graphs for musical compositions separately for pitch frequencies and time durations. For eliciting the reduction and the visual pattern recognition through the interconnections of the graph, we evaluate their construction on 1409 tracks of classical music consisting of compositions of Beethoven, Mozart, and Bach. The adjacency matrix of the network implicitly creates a statistical method for evaluation of the underlying note sequences. We can draw out the



Figure 4. Learning Curve for each of the Compositions w.r.t. Table I.

probability of the next note being played from a current note, similarly for the relative time duration. This also helps us to draw out the probability distribution across all the notes form a current note. Which allows us to compare the probability of *n*-length note sequence being present in the composition via Bayes theorem. We observe symmetries in temporal graphs across all the composers. The networks so formed gives us a visual analysis of quantifying as to how many time durations have been used by the composer. Drawing out the degree distribution, observe a power-law degree distribution for the time networks. Indicating that a small amount of time duration has been repeated for a large number of times, and a large amount of time durations has been repeated a small amount of time. This indicates the self-organizing behaviour of time networks. We further extend the study by constructing a reinforcement learning (RL) model which aims to predict the next note, with maximum accuracy given the present note, and analyze the results found for different compositions belonging to different composers.

The pitch graph proposed helps us to visualize notes and their interconnections with the other notes. This can help identify motifs via calculating flows [16]. Multiple networks can be drawn, and compared simultaneously to differentiate between each category of composition. This property may find its significance for a music therapist to investigate which harmonies and time durations influence the patient. For a music synthesizing engineer, it may help to study the rules of how the music is composed across genres, decades, and communities [10, 37, 31]. A neurologist may also draw out and conclude the effect that time durations music produces in the human brain, which may be taken up by a music therapist designing music contentfor a patient suffering from a specific disease [30, 39]. Some of the limitations of the proposed work include:

- Computational bound, wherein for allocation of the nxn matrix of temporal networks, for certain compositions, we are not able to allocate that amount of memory,
- We cannot process the commonly available mp3 format of the compositions, we require its MIDI file,
- Low resolution in temporal networks, due to a relatively greater amount of time durations present in the network as compared to the musical network, we avoid labeling the time nodes leading to a lesser perceivable information at a glance.

Music has existed and evolved with language. The construction of graphs can be used to study not only the current constituents of music but also for historical musical compositions. Various compositions have been able to stand for people's cultures, and beliefs. We could draw out factors that influenced the high social impact of the different kinds of musical compositions used in an earlier era. With advances in neurology and this formation, we can now draw out the parallel between structure of music, and its impact on our body at a granular-level [9, 23]. Music can be incorporated into daily habits, and routines to promote the well being of the society as a whole [24]. Such a study can help improve, on an individuallevel, the sounds we expose ourselves daily for our well being. In the network of the world, there are many nodes, one of them being music which is a well-connected node, which may also be identified as a hub, influencing every other node connected to it. With understanding it on a structural level, we can make sure to create a positive impact and improve the quality of our life. We observe that in nature symmetry is present in order to balance, like the two sides of a leaf balance force equally on the stem they are placed. Evidences of symmetry in music formulate a possibility of balance found there in. Dance is an infinite set of artistic movements performed on music, so in future we wish to look deeper into the structures and use dance as another form of visualization to quantify balance present in music. In addition, our future research efforts will also be directed towards posing the problem of modeling music structure as topological structure (i.e. rubber sheet geometry) and correlate with topology of complex networks.

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