An Improved Method for Instantaneous Frequency Estimation Using a Finite Order Hilbert Transformer

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***Abstract*—The present paper describes an instantaneous frequency (IF) estimation method. The instantaneous amplitude and phase can be obtained simultaneously by converting the real signal into a complex time signal using the Hilbert transform. In addition, the IF is obtained by differentiating the instantaneous phase with respect to time. However, the amplitude of the signal obtained by phase-shifting the input signal by 90-degrees is different from that of the input signal because a ripple is generated in the frequency characteristics of the finite order finite impulse response (FIR) Hilbert transformer (HT). As a result, errors are contained in the IF. In the present paper, we theoretically show that the IF obtained using the finite order HT contains harmonics. Moreover, we propose a method by which to remove harmonics using an FIR filter with the specified transmission zeros.**

I. Introduction

Sensors currently used in electronic weighing instruments are mainly electromagnetic force balance, strain gauges, and tuning forks. Tuning fork scales share the advantages of both sensors and are widely used in, e.g., the pharmaceutical, automotive industries, chemical plants and precious metal processing plants[1].

The basic principle of the tuning fork sensor is that this sensor uses a physical phenomenon where by the resonant frequency $f(t)$ of a tuning fork vibrator changes when the tension changes with weight. The scale measures this frequency change, which is then convert to a weight value. Therefore, it is necessary to estimate the value of a single frequency with high speed and high accuracy.

The frequency of a single sine wave can be measured using a frequency counter or the instantaneous frequency (IF). In the former method using a frequency counter, the reciprocal type frequency counter is commonly used. Using this method, the output becomes the irregular interval. Therefore, when digital processing is performed on a value measured using this method, a device such as an interpolation processor is required[2].

In the latter method, the IF is obtained by time-differentiating the phase of the analytic signal consisting of the original signal and its Hilbert transform [3]. Therefore, there is an advantage in that the output is generated at regular intervals. There are many different implementations of the FIR Hilbert transformer (HT) [4], [5], [6]. Pei and Shyu [4] presented a HT based on an eigenfilter, and Kollar et al. [5] presented a HT based on the least squares and the minimax criteria. Lim et al. [6] introduced a method for the synthesis of a very sharp HT using a frequency-response masking technique. These HTs can be implemented as a digital filter with finite order in actual systems. Therefore, the frequency characteristics of the obtained FIR HT contain ripples. As a result, the amplitude of the signal obtained by phase-shifting the input signal by 90-degrees using the FIR HT is different from that of the input signal. Therefore, since the estimated IF includes harmonic frequency components, an accurate IF cannot be obtained. To obtain a more accurate IF, the order of the filter may be increased to reduce ripples in the HT. However, increasing the filter order increases the delay time. This becomes a problem in applications where fast IF estimation is required.

In this paper, we propose a highly accurate IF estimation method using FIR filter with the specified transmission zeros. A harmonic frequency component is included in the IF obtained using a finite order FIR HT. Then, it is shown that an accurate estimate of the IF can be obtained by removing the harmonic frequency component with FIR filter with the specified transmission zeros. Finally, the effectiveness of the proposed method is shown in simulations.

II. IF estimation using finite order Hilbert transformer

The HT is a digital filter that changes the phase of the input signal by $\pi/2$ without changing the amplitude of the input signal. The ideal frequency response of the HT is given by

$$D(e^{j\omega}) = \begin{cases} -j & (0 < \omega < \pi) \\ 0 & (\omega = 0) \\ j & (-\pi < \omega < 0). \end{cases} \tag{1}$$

When the HT is realized as an N-order linear phase FIR filter, the zero phase amplitude characteristic of its frequency response $H_0(e^{j\omega})$ is expressed as

$$H_0(e^{j\omega}) = \begin{cases} 2\displaystyle\sum_{n=0}^{\frac{N-2}{2}} h(n)\sin\{(n-\frac{N}{2})\omega\} & (N:\text{even}) \\ 2\displaystyle\sum_{n=0}^{\frac{N-1}{2}} h(n)\sin\{(n-\frac{N}{2})\omega\} & (N:\text{odd}). \end{cases} \tag{2}$$

In this paper, the filter coefficient, $h(n)$, is obtained using the Remez algorithm [7].

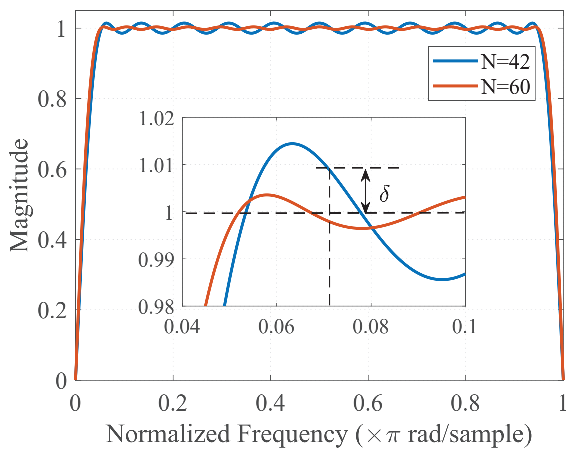

Fig. 1. Amplitude characteristic of the Hilbert transformer of finite order.

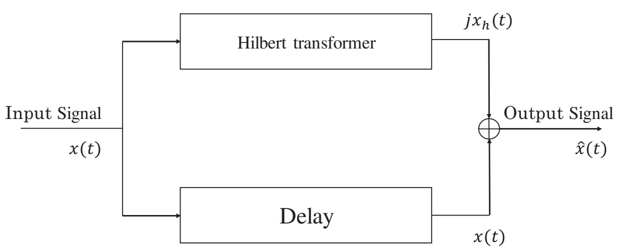

Fig. 2. System diagram of the analytic signal generated using the Hilbert transformer.

Next, we consider an HT of order $N = 42$ or 60, low passband edge normalized frequency is 0.05, and high passband edge normalized frequency is 0.95. The amplitude characteristic of the HT obtained for each order is shown in Fig. 1. As shown in Fig. 1, increasing the filter order reduces the size of the ripple. The HT of the input signal $x(t)$ produces a 90-degrees phase shifted signal, $x_h(t)$. Thus, the complex signal $\hat{x}(t)$ obtained by the system shown in Fig. 2 becomes

$$\hat{x}(t) = x(t) + jx_h(t). \tag{3}$$

The angle of the complex signal $\hat{x}(t)$ in (3) has the following instantaneous phase,

$$\phi(t) = \arctan \frac{x_h(t)}{x(t)}. \tag{4}$$

Moreover, the IF obtained by the time derivative of the phase shown in (4) is defined as

$$\tilde{f}(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}. \tag{5}$$

As a specific example, the input signal and the 90-degrees phase shifted signal obtained for each order of the HT are shown in Fig. 3 and an enlarged view is shown in Fig. 4. Here, the input signal is a sine wave with an amplitude of 1 and a frequency of 1,800 Hz. As shown in Fig. 4, the signal after the HT has a different amplitude than the original signal due to the magnitude of the HT ripple at the frequency of the input signal. The IF estimated using the HT in Fig. 1 is shown in Fig. 5. Figure 5 shows that the estimated IF oscillates around the true frequency. Since this vibration component is the error due to ripple, the higher the order of the HT, the higher the estimation accuracy.

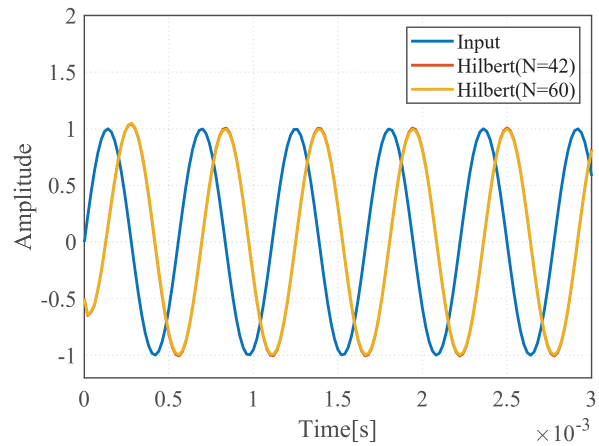

Fig. 3. Input signal and HT signal.

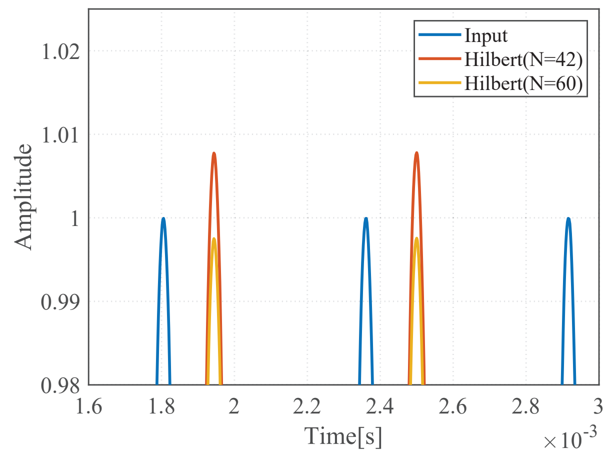

Fig. 4. Enlarged view of Fig. 3

Next, we consider in detail the estimated IF. We denote the input signal in Fig. 2 as

$$x(t) = A \sin \omega_1 t, \tag{6}$$

where A and ω_1 are the amplitude and angular frequency, respectively. If the difference in amplitude between the Hilbert transform signal and the input signal is $A\delta$ because of the finite order HT, then the Hilbert transform signal, $x_h(t)$, becomes

$$x_h(t) = -A(1-\delta) \times \cos \omega_1 t. \tag{7}$$

When (6) and (7) are substituted into (4), the instantaneous phase is

$$\phi(t) = \arctan\left(\frac{-(1-\delta)\cos\omega_1 t}{\sin\omega_1 t}\right). \tag{8}$$

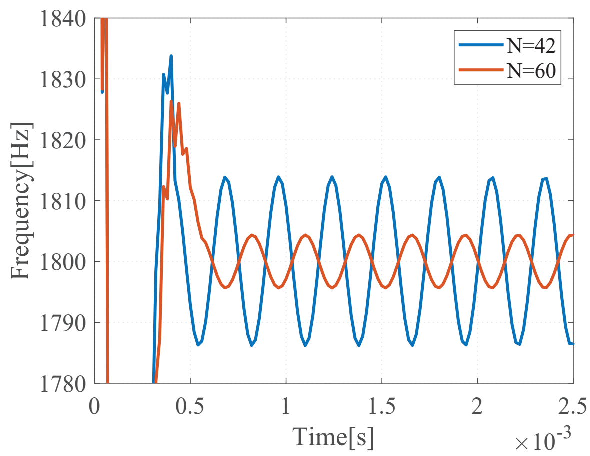

Fig. 5. Instantaneous frequency (IF) estimated using the finite order Hilbert transformer.

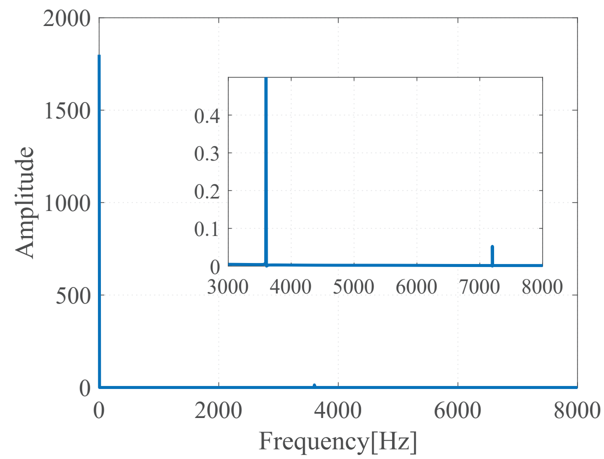

Fig. 6. Fourier spectrum of the IF ($N = 42$).

The instantaneous angular frequency obtained by the time derivative of (8) is

$$\begin{aligned}\tilde{\omega}(t) &= \frac{d\phi(t)}{dt} \\ &= \frac{2(1-\delta)\omega_1}{1+(1-\delta)^2+[(1-\delta)^2-1]\cos(2\omega_1 t)}. \end{aligned} \tag{9}$$

When $\delta = 0$, the instantaneous angular frequency is equal to the input angular frequency ω_1. However, when $\delta \neq 0$, the instantaneous angular frequency is not constant. Therefore, the estimated IF oscillates around the frequency of the input signal as shown in Fig. 5.

In order to analyze the vibration component, we consider the Fourier series expansion of $\tilde{\omega}(t)$;

$$\tilde{\omega}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2\omega_1 n t), \tag{10}$$

where

$$a_n = \frac{4\omega_1}{\pi}\int_0^{\frac{\pi}{2\omega_1}} \tilde{\omega}(t)\cos(2\omega_1 n t)dt. \tag{11}$$

Here, the Fourier coefficients a_n are

$$\begin{aligned} a_0 &= 2\omega_1 \\ a_1 &= \frac{2\omega_1\delta^2}{1-(1-\delta)^2} \\ a_2 &= \frac{2\omega_1\delta^4}{\{1-(1-\delta)^2\}^2} \\ &\cdots \end{aligned} \tag{12}$$

Therefore, the true value of the instantaneous angular frequency consists of a DC component and vibration components that are harmonics of even multiples of the original angular frequency. Thus, the estimated IF also has vibration components that are harmonics of even multiples of the frequency of the input signal.

In order to confirm the above statement, the Fourier spectrum of the estimated IF in Fig. 5 is shown in Fig. 6. The IF has a component with a frequency of $3,600\,\text{Hz}$, which is twice the frequency of the input signal, and another component with a frequency of $7,200\,\text{Hz}$, which is four times the frequency of the input signal. Therefore, both harmonic components can be removed, an accurate IF can be obtained even with a finite order HT.

III. FIR FILTER WITH THE SPECIFIED TRANSMISSION ZEROS

In this paper, we use FIR filter with transmission zeros to remove vibration components. The second order FIR filter with transmission zeros having a transfer function is written as

$$H_p(z) = 1 - 2\cos\left(2\pi f_p/f_s\right) z^{-1} + z^{-2} \tag{13}$$

and can remove the component of frequency f_p from an input signal[8]. In (13), f_p is the transmission zero frequency and f_s is the sampling frequency. It is clear from Fig. 6 that the estimation accuracy improves as the transmission zeros increase. Therefore, in this paper, we set a minimum value for the maximum number of transmission zeros that can be included in $f_s/2$ within the expected input frequency range. For example, if $f_s = 50\,\text{kHz}$ and the input frequency range is from $1,600\,\text{Hz}$ to $2,000\,\text{Hz}$, the number of transmission zeros that can be placed in the range below $f_s/2$ is seven at $1,600\,\text{Hz}$ and six at $2,000\,\text{Hz}$, therefore an FIR filter with six transmission zeros is designed. Figure 7 shows the amplitude characteristic of FIR filter with six transmission zeros when the transmission zero frequencies are even multiples of $1,600\,\text{Hz}$. It's clear from Fig. 7 that the signal is amplified at higher frequencies This is a problem when the high frequencies contain noise. Therefore, the amount of attenuation must be secured in all frequency bands by designing a correction filter[9]. The amplitude characteristics of a 40th order filter consisting of a filter with six transmission zeros and 28th

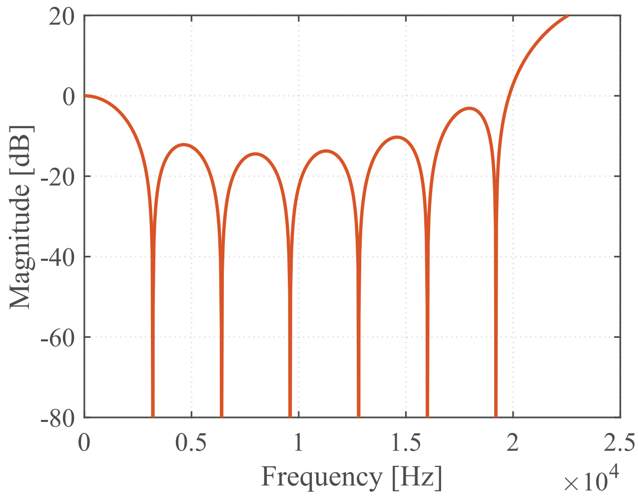

Fig. 7. Amplitude characteristic of FIR filter with six transmission zeros.

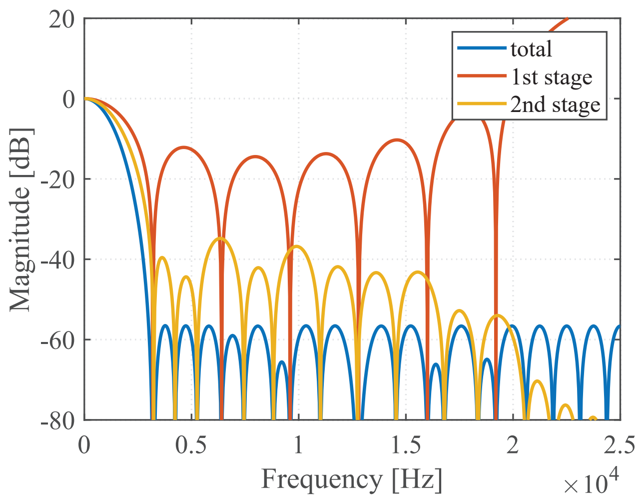

Fig. 8. FIR low-pass filter with six transmission zeros.

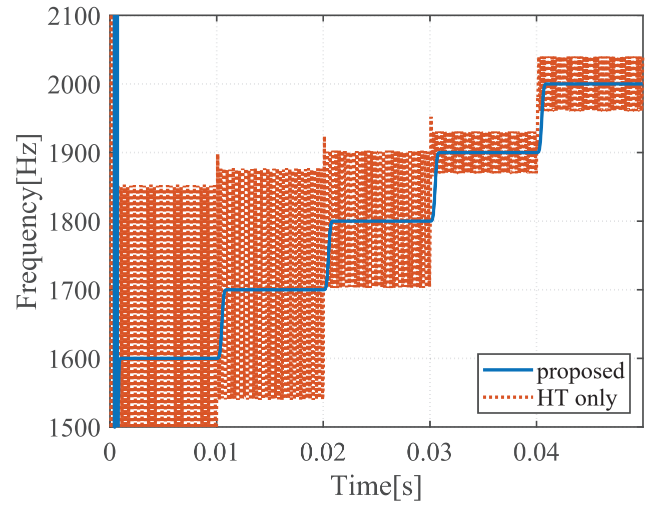

Fig. 9. IF obtained by the proposed method.

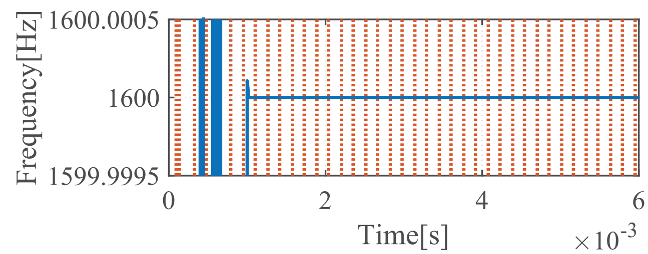

Fig. 10. Enlarged view of Fig. 9 in the $1,600\,\mathrm{Hz}$ part.

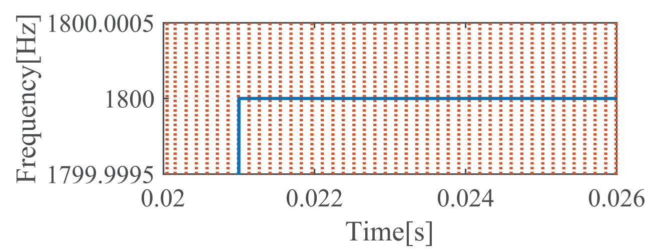

Fig. 11. Enlarged view of Fig. 9 in the $1,800\,\mathrm{Hz}$ part.

order filters to compensate for them are shown in Fig. 8. Furthermore, the stopband edge frequency is set to $3,000\,\mathrm{Hz}$.

IV. Simulation

We show that the estimation accuracy of the IF can be improved by removing these vibration components using FIR filter with transmission zeros. In this simulation, the input signal is a sine wave with an amplitude of 1 and a discontinuous frequency from $1,600\,\mathrm{Hz}$ to $2,000\,\mathrm{Hz}$, sampled at a sampling frequency of $50\,\mathrm{kHz}$, assuming the output from the tuning fork sensor. The low and high passband edge frequencies of HT with 10th order are $1.25\,\mathrm{kHz}$ and $27.25\,\mathrm{kHz}$, respectively. We use a 40th order FIR filter shown in Fig. 8 to remove the vibration component. Here, the transmission zero frequencies were even multiples of the previous IF with the vibration component removed, and the filter was redesigned for each sample.

Figure 9 shows the IF estimated using the 10th order HT and the IF after removing the vibration components by the proposed method. Furthermore, an enlarged view of the $1,600\,\mathrm{Hz}$ and $1,800\,\mathrm{Hz}$ are shown in Fig. 10 and 11. These results are also shown in Fig. 9–11 show that the proposed method enables accurate IF estimation.

Next, the performance under the noise environment is evaluated. In order to show the usefulness of the proposed method, we compare the results using the proposed method with those using a high order HT. In order to get closer to the conditions, a low-pass filter designed using the Remez algorithm was used in the case of high order HT. The order of the low-pass filter was set to 40th order as in the proposed method. The order of HT was increased until the accuracy of

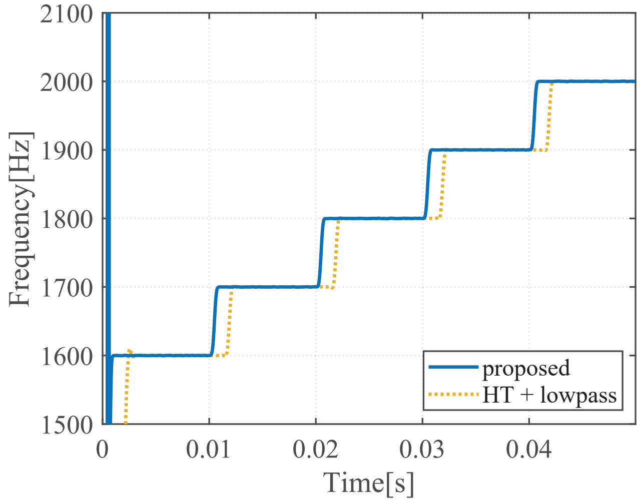

Fig. 12. Results in noisy environments.

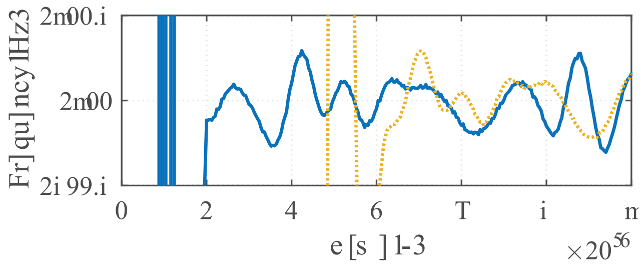

Fig. 13. Enlarged view of Fig. 9 in the 1, 600 Hz part.

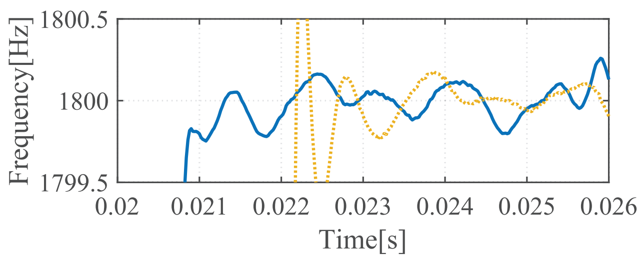

Fig. 14. Enlarged view of Fig. 9 in the 1, 800 Hz part.

the proposed method was achieved. The maximum value of variance of each frequency was used to compare the accuracy, and the order of HT required to outperform the proposed method was 150th order. Figure 12–14 shows the results of adding 60 dB of white noise to the input signal. The values of the variance at each frequency are shown in Tab. I. From these results, we can confirm that fast and accurate IF estimation is possible even in noisy environments.

TABLE I
VARIANCE AT EACH FREQUENCY

method	**variance**				
	1,600 Hz	**1,700** Hz	**1,800** Hz	**1,900** Hz	**2,000** Hz
proposed	0.0184	0.0227	0.0158	0.0120	0.0199
HT+lowpass	0.0210	0.0178	0.0163	0.0107	0.0181

V. Conclusion

In this paper, we theoretically showed that the IF obtained using an HT with a finite order contains harmonics. Moreover, we proposed a method to remove harmonics using an FIR filter with the specified transmission zeros. By removing the harmonics, an example was used to demonstrate that a high precision IF could be obtained even with a low order HT.

In the future, the use of a variable filter will eliminate the need to redesign the filter for each sample.

References

[1] N. Shinozaki, K. Okamoto, M. Ikeshima, T. Terunuma, and K. Naito, “Per-formance of tuning fork load cell,” *oiml bulletin*, pp. 20–27, 2015.

[2] S. Johansson and T. Horita, “Frequency Counter,” *The Review of Laser Engineering*, vol. 39, no. 10, pp. 775–779, 2011.

[3] H. Suzuki, F. Ma, H. Izumi, O. Yamazaki, S. Okawa, and K. Kido, “Instantaneous frequencies of signals obtained by the analytic signal method,” *Acoustical Science and Technology*, vol. 27, no. 3, pp. 163–170, 2006.

[4] P. Soo-Chang and S. Jong-Jy, “Design of FIR Hilbert transformers and differentiators by eigenfilter,” *IEEE Transactions on Circuits and Systems*, vol. 35, no. 11, pp. 1457–1461, Nov. 1988.

[5] I. Kollar, R. Pintelon, and J. Schoukens, “Optimal FIR and IIR Hilbert transformer design via LS and minimax fitting,” *IEEE Transactions on Instrumentation and Measurement*, vol. 39, no. 6, pp. 847–852, Dec. 1990.

[6] Y. C. Lim, Y. J. Yu, and T. Saramaki, “Optimum masking levels and coef-ficient sparseness for Hilbert transformers and half-band filters designed using the frequency-response masking technique,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 52, no. 11, pp. 2444–2453, Nov. 2005.

[7] J. McClellan, T. Parks, and L. Rabiner, “A computer program for designing optimum FIR linear phase digital filters,” *IEEE Transactions on Audio and Electroacoustics*, vol. 21, no. 6, pp. 506–526, Dec. 1973.

[8] Y. Tadokoro and K. Abe, “Notch Fourier transform,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, no. 9, pp. 1282–1288, Sep. 1987.

[9] N. Aikawa and M. Sato, “A new structure of prefilters for FIR digital filters,” *Electronics and Communications in Japan (Part III: Fundamental Electronic Science)*, vol. 72, no. 5, pp. 94–101, 1989.