A Fractional Integrator Based Novel Detector for Weak Signal Detection with Watermark Application

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***Abstract*—The detection of a weak signal from noisy data is an important task in many signal processing applications such as radar communication, biomedical engineering *etc.* However, the amplitude of the known signal plays a big role in terms of complexity and performance of the detector. In other words, detection of the weak signal is a big challenge. Here, we investigate approximated fractional integrator (AFI) based detector.**

The proposed method has been employed for detection of DC signal which is present in the Gaussian noise. Our proposed method has been compared with some state-of-the-art methods in terms of probability of detection (P_D) for a constant value of probability of false alarm (P_{FA}). The P_D has been plotted for varying signal-to-noise ratio (SNR) at a constant value of P_{FA}. Furthermore, we apply the proposed method for watermark application. The outcomes of the proposed method are convincing and it suggests that the proposed method works better or comparable to some state-of-the-art methods.

I. Introduction

Signal processing techniques play an important role in various applications particularly in solving real-life problems. When the receiver receives the signal, the strength of the received signal depends upon the surrounding environment and it may degrade as it is affected by so many noise sources at different stages. The detection of the known low-frequency weak signal is a scientific challenge in many science and engineering fields such as communication, signal and image processing [1] *etc*. It is sometimes quite difficult to detect the original signal present in noise ($A <<< \sigma$), where A and σ are the magnitude of the known signal and standard deviation of noise respectively. Therefore, for detecting the known weak signal in the presence of noise, a novel detector is required to be investigated.

In recent years, the fractional operator has been advocated as an important mathematical tool [2], [3], [4] and it has been utilized in many signal [5], [6] and image processing applications. In general, the fractional operator has explored from an integer step to fractional step. Depending upon the value of fractional order, fractional operator can be divided into fractional differential operator ($0 < q < 1$) and fractional integrator (approximated fractional integrator (AFI)) ($-1 < q < 0$) [7], [8]. It has been demonstrated by scientific findings that fractional order approach is best suited to many natural phenomena discussed in Ref. [9]. Applying fractional operator for signal analyzing and processing [10], [11] and an image processing [12] is a challenging task. There are evidences which show that fractional order based algorithms are powerful approaches in many applications [13] due to its non-linear behaviour. In addition, the fractional operator has also been used in enhancing complex fractal-like texture details non-linearly.

Key contribution of the paper: The key contribution of the paper is to use the AFI for weak signal detection. To the authors' knowledge, AFI has not been previously employed for signal detection in literature. The negative fractional order plays a key role in boosting the signal and suppressing the noise. Thereafter, a watermark detection application has also been exhibited using the concept of AFI.

Organization of the paper: Section II deals with the basic mathematics of detection theory and fractional integrator. The proposed method is presented in Section III. However, the proposed theory has been applied in watermark detection and has been discussed in Section IV. Results have been discussed in Section V. Finally, the conclusion has been presented in Section VI.

II. Basic Mathematics

In this section, we review the basic theory of signal detection and fractional integrator in brief.

A. Detection Theory

The detection of a known weak signal present in noisy data is a practical problem for different applications. The detection problem can be modelled by the discrete equation which is given below.

$$\begin{cases} H_0 : x(i) & = w(i) \\ H_1 : x(i) & = s(i) + w(i), \end{cases} \tag{1}$$

where $i = 0, \ldots, (N-1)$ and s is known signal. Here,$w(i)$ represents independent and identically distributed (iid) Gaussian noise. $x(i)$ is put under the category of H_0 or H_1, when it is compared with a threshold.

B. Fractional Integrator

The fractional integral of function $f(t) \in C(I),\ I = [0, T]$ with order q is defined as follows [14].

$$I^q_{0,t} f(t) = \frac{1}{\Gamma(-q)} \int_0^t (t-r)^{-q-1} f(r) dr,\ q < 0 \tag{2}$$

where r is dummy variable and Γ is gamma function. Eq. 2 is approximated by the following form

$$\left[I_{0,t}^{q} f(t)\right]_{t=t_n} \approx \sum_{k=0}^{n-1} b_{n-k-1} f(t_k), \tag{3}$$

where

$$b_k = \frac{(\Delta t)^{-q}}{\Gamma(-q+1)} \left[(k+1)^{-q} - k^{-q}\right], \tag{4}$$

$k = 0, 1, \ldots, (N-1)$. Eq. 3 is called the fractional rectangular formula for the fractional integrator and is also termed as AFI. In our case, we assume $\Delta t = 1$.

Remark 1: Convergence order of this method discussed in Eq. 3 is $O(\Delta t)$ for $q < 0$. If $q = -1$, this formula reduces to the classical integral formula.

Remark 2: Eq. 3 is valid for all integer or fractional order for $q > 0$ or $q < 0$.

Remark 3: The AFI behaves as a non-linear low pass filter.

Remark 4: The coefficients of Eq. 4 are used to design the filter for a particular value of fractional order q $(-1 < q < 0)$.

III. Proposed Method

Here, we discuss the working of our proposed method, which is shown in Fig. 1.

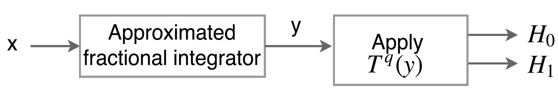


Fig. 1: Flow chart of the proposed method.

When the AFI based filter is convolved with x, it gives y which is written as follows.

$$y(i) = I^q \begin{cases} w(i), & No\ signal\ present \\ s(i) + w(i), & Signal\ present. \end{cases} \tag{5}$$

Here, in this paper, we assume w and s as Gaussian noise and some known signal respectively. Applying Neyman-Pearson criteria, it produces the following test statistics.

$$T^q(y) = \frac{1}{N} \sum_{i=0}^{N-1} s[i]y[i] \tag{6}$$

Under different hypotheses, the distribution of the test statistics for $s(i) = A$ can be written as follows.

$$T^q(y) \cong \begin{cases} N(0, \frac{\sigma_{new}^2}{N}) & under \quad H_0 \\ N(A\mu_{new}, \frac{\sigma_{new}^2}{N}), & under \quad H_1 \end{cases} \tag{7}$$

where

$$\mu_{new} = \sum_{i=0}^{N-1} (-1)^i \left({}^qC_i\right) \tag{8}$$

and

$$\sigma_{new}^2 = \sigma^2 \sum_{i=0}^{N-1} (-1)^{(2i)} \left({}^qC_i\right)^2. \tag{9}$$

The probability of false alarm is

$$P_{FA} = Prob\left(T(y) > \gamma'; H_0\right) \tag{10}$$

$$P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\frac{\sigma_{new}^2}{N}}}\right) \tag{11}$$

Similarly, the probability of detection (P_D) can be written as follows.

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{NA^2\mu_{new}^2}{\sigma_{new}^2}}\right). \tag{12}$$

The Fig. 2, Fig. 3 and Fig. 4 show the performance of the proposed detector and a comparison with some known methods.

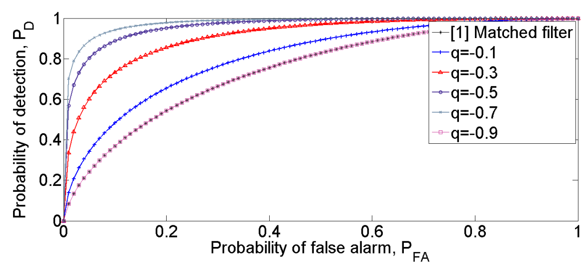


Fig. 2: Probability of detection, P_D versus P_{FA} at $A = 0.1$, $N = 100$ and $\sigma = 1$.

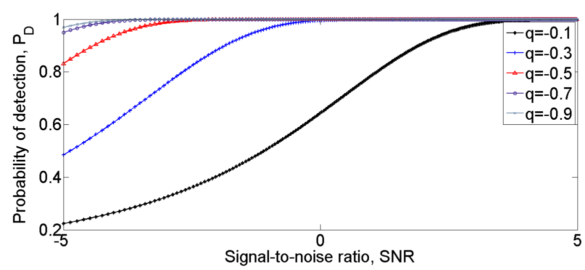


Fig. 3: Probability of detection, P_D versus signal-to-noise ratio at $P_{FA} = 0.1$, N=100 and $\sigma = 1$.

IV. Watermarking Application

We traditionally store our data such as text, audio, still images, animation, video, or interactive content forms, music, *etc.,* in the storage like compact disc or films. Technological revolution helps in the digitization of the data in the sequence of 1s and 0s, which ultimately helps in storage, distribution, processing *etc.* But at the same time, the technological advancement has also added us a brand problem due to illegal reproduction and redistribution. The watermarking [15] ensures the communication between two authorized user *i.e.,* the sender and the receiver. A watermark is defined to be something that is added into the cover materials.

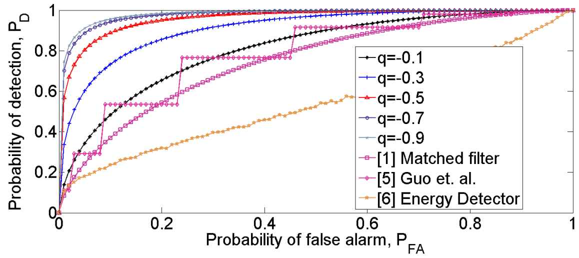


Fig. 4: Comparison study of the different methods. Probability of detection, P_D is drawn against P_{FA} at $A = 0.1$, $N = 100$ and $\sigma = 1$.

The important steps of the digital watermarking are (a) Embedding and (b) Extraction. The embedding [16] signifies withholding any private signature (audio, picture, signal or any random sequence) into the original image. Actually, embedding is done in three ways (a) Spatial domain watermarking (b) Transform domain watermarking (c) Hybrid domain watermarking. Here, we embed in the spatial domain. In digital image watermarking method, the watermark signal is inserted into the pixel values directly in the spatial domain or it can be added in the transform domain [17]. Mathematically, it is written as follows.

$$Y_{wm} = L_{ori} + \alpha_i W_i, \tag{13}$$

where 'L_{ori}' shows the 'Lena' image, 'α_i' exhibits the strength of the watermark signal and W_i represents the watermark signal. Here, in our experiment, we have embedded in the spatial domain and obtain the watermarked image. It can be observed in Fig. 5a, Fig. 5b and Fig. 5c. Fig. 5c is attacked by Gaussian noise to get Fig. 5d. Let us assume this image as Y_A. Thereafter, noisy watermark image Z_i can be obtained by the following formula.

$$Z_i = Y_A - L_{ori}, \tag{14}$$

where Z_i is noisy watermark signal. During watermark extraction process, we first formulate the watermark extraction problem as a binary signal detection problem. Watermark detection means that we have to find the absence or presence of the signal in noisy watermark image Z_i. The detection of watermark is mathematically modelled as follows.

$$\begin{cases} H_0 : Z_i & = 0 + W_i \\ H_1 : Z_i & = 1 + W_i, \end{cases} \tag{15}$$

where 0 and 1 represent the known signal and W_i indicates the noise which is already incorporated in the signal. Here, Z_i is attacked watermark image. We model this detection problem for 0 and 1 because the watermark image carries 0 and 1 only. Now, we calculate the threshold value which causes the correct detection of the weak signal *i.e.,* detection is carried out in the Neyman-Pearson criteria. The threshold value [18] is given as follows.

$$th = erfc^{-1}(2P_{FA})\sqrt{2\sigma_{new}^2} + \mu_0, \tag{16}$$

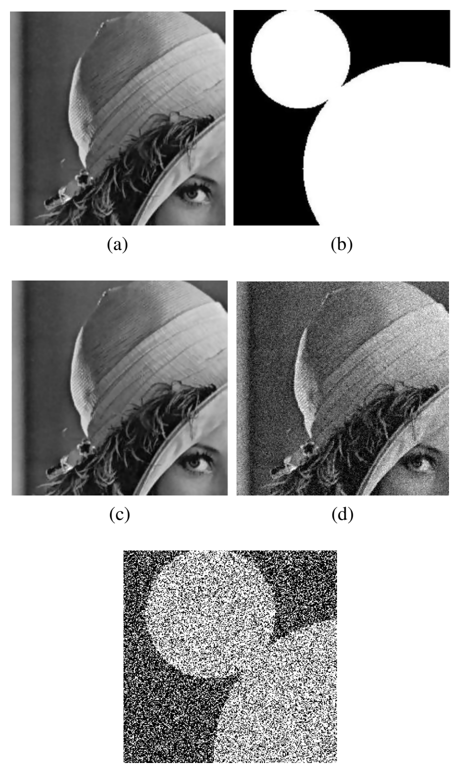
where P_{FA}, σ_{new}^2 and μ_0 are probability of false alarm (which is actually known constant value), variance after AFI stage and mean after AFI stage under H_0 hypothesis respectively. The σ_{new} is approximated by Eq. 9.

V. Result & Description

Here, the results of the proposed method for detection theory and watermark application have been discussed. For simulation, we chose MATLAB 2014b.

A. Results of detection theory

Fig. 2 is the receiver operating characteristic (ROC) curve of the proposed filter at different orders, q at $A = 0.1$, $N = 100$ and $\sigma = 1$. $q = 0$ represents the MF [1]. The outcomes suggest that our proposed method with $q = -0.9$ performs better than all other orders. Similarly, the Fig. 3 is the plot of P_D for different value of SNR ($SNR = 20 \log \frac{A^2}{\sigma^2}$). The outcomes suggest that our proposed method performs better than the optimum matched filter detector [1]. However, the comparison of the proposed method with state-of-the-art methods is shown in Fig. 4. Our proposed method works far better when $q < -0.1$, however, for $q = -0.1$, it works better than MF [1] and ED [6] but suffers with compare to Guo et al. [5].



(a) (b) (c) (d) (e)

Fig. 5: The images are (a) Original, Lena (L_{ori})(b) Watermark, W_i (c) Watermarked, Y_{wm} (d) Attacked Watermarked (e) Extracted using AFI with $q = 0$. The dimension of L_{ori} and W_i are 256×256. The $\alpha = 0.1$ has been chosen.

TABLE I: MEAN SQUARE ERROR (MSE) AT DIFFERENT NEGATIVE ORDERS FOR DISTINCT ATTACKS (DIFFERENT VALUES OF VARIANCE OF GAUSSIAN NOISE). CONSIDER $P_{FA} = 0.01$.

MSE at $P_{FA} = 0.01$										
Attack (var), q ($\rightarrow$)	0	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9
Gaussian (0.1)	0.3891	0.3176	0.0124	0.0053	0.0040	0.0036	0.0034	0.0034	0.0035	0.0036
Gaussian (0.5)	0.4023	0.2998	0.1806	0.0969	0.0547	0.0334	0.0234	0.0183	0.0159	0.0145
Gaussian (0.9)	0.4096	0.3178	0.2459	0.1890	0.1492	0.1241	0.1089	0.0987	0.0923	0.0883

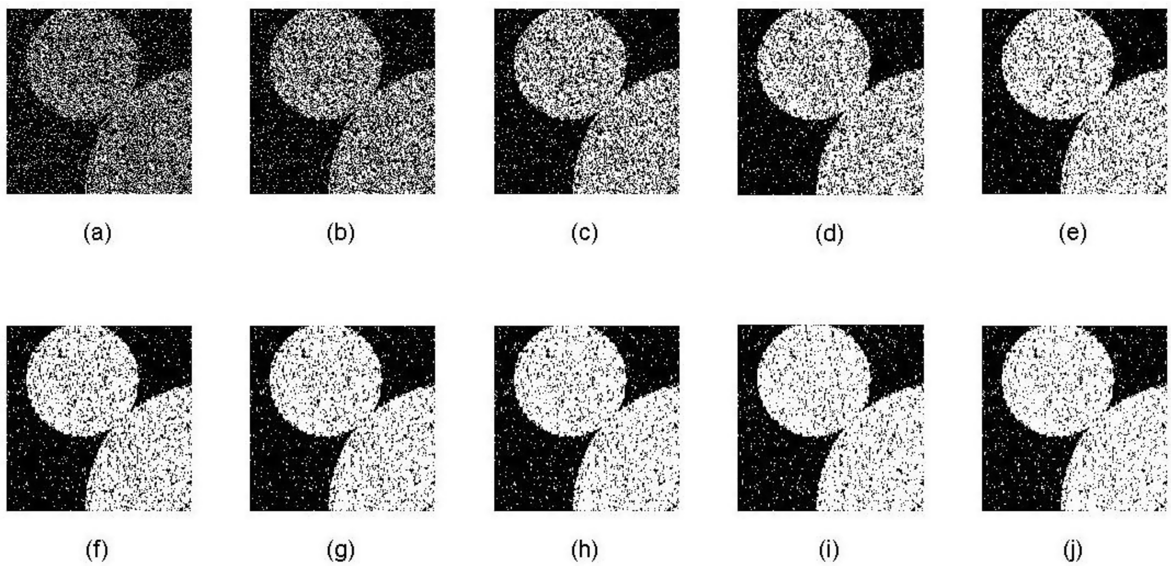


Fig. 6: Attack of ($var = 0.9$). Images recovered at different order, q (a) 0 (b) -0.1 (c) -0.2 (d) -0.3 (e) -0.4 (f) -0.5 (g) -0.6 (h) -0.7 (i) -0.8 (j) -0.9

B. Results of watermark detection

For the detection of the watermark signal [19], we apply the fractional integrator based filter [20]. Thereafter, we need a threshold th, which can be calculated using the formula discussed in Eq. 16. We recover the watermark signal with the help of AFI based filter and attacked watermarked image. It has been shown in Fig. 6. For the comparison, we show the mean square error (MSE).

In Table I, we can see how the MSE is decreasing by the use of fractional order integrator. We consider $var = 0.1,\ 0.5,\ 0.9$ for the attack of Gaussian noise.

The $MSE = 0.0040$ at $q = -0.4$, $MSE = 0.0036$ at $q = -0.5$ and $MSE = 0.0034$ at $q = -0.6$ are obtained when the watermarked image is attacked with Gaussian noise with $var = 0.1$. It clearly mentions that detection capability of the proposed detector at the different fractional order, q. However, MSE gets reduced from 0.4023 at $q = 0$ to 0.0145 at $q = -0.9$, when the attack of $var = 0.5$ is considered. It clearly exhibits that as the order goes to -0.9, it produces the least MSE. At the same time, if we observe the MSE at some constant value of q, it results in more error as variance increases. It can be studied for the same column in Table I. It suggests that MSE is more for the high value of attack (var) at constant q.

Fig. 6 is the set of recovered images at distinct fractional orders (q) when the watermark image is attacked by Gaussian noise ($var = 0.9$). Fig 6(j) gives the image with the least MSE as supported by Table I *i.e.,* $MSE = 0.0883$ at $q = -0.9$ when the watermarked image is attacked by $var = 0.9$. The image Fig. 6(j) offers much visibility among all, as supported by the Table I. This justifies that the performance of the proposed detector gets boosted, as the value of MSE gets lowered down when fractional order (q) goes towards $-0,9$. It concludes that the AFI based filter enhances the detection performance.

VI. Conclusion

This paper proposes the weak signal detector which is based on low pass fractional integrator filter. The proposed results suggest that the proposed detector outperforms than the state-of-the-art methods. For $A = 0.1$, $N = 100$, $\sigma = 1$, the increment in $P_D = (0.9 - 0.4) = 0.5$ at $P_{FA} = 0.1$ and $q = -0.9$. The proposed method has also been employed for watermark detection. The significant decrement in MSE shows the robustness of the proposed detector. Our future work is to design the detector which is robust and performs well in non-Gaussian noise such as Gaussian mixed, generalized Gaussian, Laplacian noise *etc.*

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