MOCAP signal interpolation using low-rank matrix recovery

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Abstract—We propose a new regularization method for 3D MOCAP (Motion Capture) signal restoration. As a MOCAP signal restoration method, methods considering low-rankness and smoothness or Truncated Nuclear Norm (TrNN) for lowrank approximation have been proposed. However, with these methods, the smoothness in the time direction of the MOCAP signal cannot be sufficiently restored. In this study, from the a priori knowledge that many of human movements involve smooth acceleration/deceleration and the a priori knowledge that the acceleration/deceleration patterns are similar among related joints, we introduce the regularization based on the second-order differentiation in the time direction. In numerical experiments, we apply this method to MOCAP signal restoration with continuous missing in the time direction and show that the performance is superior to the conventional methods.

I. INTRODUCTION

Motion capture (MOCAP) technology for acquisition and analysis of human joint information is widely used in various fields such as computer animation, movie production, virtual reality and medical rehabilitation [1], [2], [3]. At the stage of capturing actions, there is a problem that loss of acquired data occurs even with specialized equipments. This defect is due to occlusion and marker ambiguity. The problem becomes more serious when simpler MOCAP equipment are used, and it is indispensable to restore missing values of acquired data [4], [5]. To address the problem, a number of deficit interpolation methods have been proposed [6], [7], [8], [9].

MOCAP signal interpolation can be achieved by completing a matrix consisting of data of each joint coordinate. In recent years, a method using nuclear norm regularization, which is used as a matrix completion method, has been proposed [2], [5], [11], [12], [13]. In these methods, by using the lowrank characteristic of the MOCAP signal, noise removal is performed and at the same time, the values of missing markers are estimated. Candes et al. [14] have approximated the low rankness by solving the nuclear norm minimization problem and proved that it can be restored accurately from only a few observation entries. Based on this, Lai et al. [10] applied the nuclear norm minimization shown in the following equation to restore the loss of MOCAP data.

$$\min_{\mathbf{x}} ||\mathbf{X}||_* \quad \text{s.t.} \quad \mathbf{X}_{\Omega} = \mathbf{M}_{\Omega}, \tag{1}$$

where $|| \cdot ||_*$ is the nuclear norm of a matrix, $\mathbf{M} \in \mathbb{R}^{m \times n}$ is incomplete observed motion data, where each row of \mathbf{M} corresponds to each marker. Ω is a set of data positions observed without any loss, and thus $\mathbf{X}_{\Omega} = \mathbf{M}_{\Omega}$ is a constraint to save data other than the missing parts.

Furthermore, the following optimization problem was proposed in [12].

$$\min_{\mathbf{X},\mathbf{S}} ||\mathbf{X}||_* + \lambda ||\mathbf{S}||_1 + \frac{\mu}{2} \Theta(\mathbf{X}) \quad \text{s.t.} \quad (\mathbf{X} + \mathbf{S})_{\Omega} = \mathbf{M}_{\Omega}, \ (2)$$

where S is a sparse component including noise and outliers included in the observed data, and $\Theta(\cdot)$ is a smoothing term with respect to the time direction.

Hu et al. [17] pointed out the disadvantage of a method of uniformly reducing the sum of the singular values of the MOCAP data and found that the nuclear norm is truncated, which is approximated as the sum of $\min(m, n) - r$ singular values, and they proposed the following model with truncated nuclear norm (TrNN).

$$\min_{\mathbf{X},\mathbf{S}} ||\mathbf{X}||_r + \lambda ||\mathbf{S}||_1 + \frac{\mu}{2} \Theta(\mathbf{X}) \quad \text{s.t.} \quad (\mathbf{X} + \mathbf{S})_{\Omega} = \mathbf{M}_{\Omega}, \quad (3)$$

where $||\mathbf{X}||_r = \sum_{i=r+1}^{\min(m,n)} \sigma_i(\mathbf{X})$ is the TrNN for $\mathbf{X} \in \mathbb{R}^{m \times n}$, and $\sigma_i(\mathbf{X})$ represents the *i*-th largest singular value of *X*. Since information possessed by MOCAP data can be represented by a small fraction of singular values, TrNN enables more effective restoration.

It is one of the important a priori information in MO-CAP data restoration that human motion has smooth acceleration/deceleration and similarity between related joints of the pattern is large, but in the above conventional methods, this issue is not addressed. Depending on the design of the smoothing term in (3), the acceleration/deceleration pattern of motion may be destroyed due to smoothing. In such a case, unnaturalness may occur in the operation of restored data.

In this research, we introduce the second-order differential regularization to TrNN for a MOCAP data restoration task. By this regularization, it is possible to restore the missing portion while considering the acceleration/deceleration information compared with the conventional method. Furthermore, while in the conventional method, the MOCAP data is directly approximated by a low-rank matrix, but in the proposed method, the low-rank approximation is performed simultaneously with the second-order regularization. This makes it possible to estimate the defective part from the viewpoint of the acceleration/deceleration pattern and achieves more accurate restoration. The convex optimization problem using this regularization can be solved by the alternate direction multiplier method (ADMM). In the experiment, we take the MOCAP signal restoration problem using two kinds of missing methods as an example and confirm its performance.

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In Section II, we describe the details of the proposed method and the solution in the optimization problem. In Section III, we restore the MOCAP signal with two kinds of deficits, random deficiency and continuous missing in the time direction, and show the superiority of the proposed method. In Section IV, we summarize this research and describe its future prospects.

II. PROPOSED METHOD

A. Problem formulation

Based on the discussion in Section I, the proposed model in this study is represented by the following minimization problem,

$$\min_{\mathbf{X}} ||\mathbf{X}||_{r} + \beta_{1} ||\mathbf{D}_{1}\mathbf{X}||_{1} + \beta_{2} ||\mathbf{D}_{2}\mathbf{X}||_{*}$$
s.t. $\mathbf{X}_{\Omega} = \mathbf{M}_{\Omega},$

$$(4)$$

The column of matrix $\mathbf{X} \in \mathbb{R}^{m \times n}$ consists of the coordinate values of xyz corresponding to each marker, m is the signal length, and n is (the number of markers)×3. where $|| \cdot ||_1$ represents the ℓ_1 norm. $\mathbf{D_1}$ and $\mathbf{D_2}$ are the differential operators denoted as

$$\mathbf{D}_1 = \begin{pmatrix} 1 & -1 & & \\ & 1 & -1 & \\ & \ddots & \ddots & \\ & & 1 & -1 \end{pmatrix} \in \mathbb{R}^{m \times m},$$
$$\mathbf{D}_2 = \mathbf{D}_1 \mathbf{D}_1,$$

respectively.

As discussed in [17], since the evaluation function using TrNN is not a convex function, it is hard to find an optimal solution. By replacing the TrNN with the approximate expression used in [16], Eq.(4) can be rewritten as

$$\min_{\mathbf{X}} ||\mathbf{X}||_{*} - \operatorname{Tr}(\mathbf{A}_{l}\mathbf{X}\mathbf{B}_{l}^{\mathrm{T}}) + \beta_{1}||\mathbf{D}_{1}\mathbf{X}||_{1} + \beta_{2}||\mathbf{D}_{2}\mathbf{X}||_{*}$$
s.t. $\mathbf{X}_{\Omega} = \mathbf{M}_{\Omega},$
(5)

 \mathbf{A}_l and \mathbf{B}_l are matrices composed of the vectors in $\mathbf{U}_l = (\mathbf{u}_1, \dots, \mathbf{u}_m) \in \mathbb{R}^{m \times m}$, $\mathbf{\Sigma}_l \in \mathbb{R}^{m \times n}$, $\mathbf{V}_l = (\mathbf{v}_1, \dots, \mathbf{v}_m) \in \mathbb{R}^{n \times n}$, which is obtained by singular value decomposition $\mathbf{X}_l = \mathbf{U}_l \mathbf{\Sigma}_l \mathbf{V}_l^{\mathrm{T}}$ of \mathbf{X}_l in the *l*-th iteration as

$$\mathbf{A}_{l} = (\mathbf{u}_{1}, \dots, \mathbf{u}_{r})^{\mathrm{T}}, \mathbf{B}_{l} = (\mathbf{v}_{1}, \dots, \mathbf{v}_{r})^{\mathrm{T}}.$$
 (6)

We repeatedly compute A_l and B_l based on X_l at each iteration and solve the minimization problem (5), and thus the MOCAP data considering acceleration/deceleration information can be restored. This process is shown in Algorism 1.

B. Computation of optimal solution

Since the fact that $||\mathbf{X}||_* - \text{Tr}(\mathbf{A}_l \mathbf{X} \mathbf{B}_l^{\mathrm{T}})$ is a convex function as is indicated in [16], the objective function expressed by (5) is also a convex function. In this paper, the alternate direction multiplier method (ADMM) is adopted as an algorithm for solving the convex optimization problem. In order to convert (5) into an unconstrained optimization problem, we introduce

Algorithm 1 Iterative Scheme
Input:
$$\mathbf{M}, \Omega$$

Initialize: $\mathbf{X}_0 = \mathbf{M}_{\Omega}$
while stopping criterion is satisfied do
 $[\mathbf{U}_l, \boldsymbol{\Sigma}_l, \mathbf{V}_l] = \operatorname{svd}(\mathbf{X}_l)$
Compute $\mathbf{A}_l, \mathbf{B}_l$ as (6).
 $\mathbf{X}_{l+1} = \arg\min ||\mathbf{X}||_* - \operatorname{Tr}(\mathbf{A}_l \mathbf{X} \mathbf{B}_l^{\mathrm{T}}) + \beta_1 ||\mathbf{D}_1 \mathbf{X}||_1 + \beta_2 ||\mathbf{D}_2 \mathbf{X}||_* \quad \text{s.t.} \quad \mathbf{X}_{\Omega} = \mathbf{M}_{\Omega}$
end while
Output: $\hat{\mathbf{X}}$, the recoverd motion matrix.

an indicator function. The indicator function $\iota_{\mathbf{M}_i}$ corresponding to the box constraint of the closed convex set $\mathbf{M}_i (i \in \Omega)$, which is defined as

$$\mathbf{M}_{i}(\mathbf{X}_{i}) = \begin{cases} 0 & \mathbf{X}_{i} = \mathbf{M}_{i}, \\ +\infty & otherwise, \end{cases}$$

ι

Using this indicator function, Eq.(5) is made to be an unconstrained convex optimization problem of the following expression,

$$\min_{\mathbf{X}} ||\mathbf{X}||_{*} - \operatorname{Tr}(\mathbf{A}_{l}\mathbf{X}\mathbf{B}_{l}^{\mathrm{T}}) + \beta_{1}||\mathbf{D}_{1}\mathbf{X}||_{1} + \beta_{2}||\mathbf{D}_{2}\mathbf{X}||_{*} + \sum_{i \in \Omega} \iota_{\mathbf{M}_{i}}(\mathbf{X}_{i}).$$
(7)

Next, the auxiliary variables are defined as $\mathbf{W}_1 := \mathbf{X}$, $\mathbf{W}_2 := \mathbf{X}$, $\mathbf{W}_3 := \mathbf{D}_1 \mathbf{X}$, $\mathbf{W}_4 := \mathbf{D}_2 \mathbf{X}$, $\mathbf{W} := [\mathbf{W}_1^T, \mathbf{W}_2^T, \mathbf{W}_3^T, \mathbf{W}_4^T]^T$, and each term is separated, it can be expressed as

$$g(\mathbf{W}) = ||\mathbf{W}_1||_* - \operatorname{Tr}(\mathbf{A}_l \mathbf{W}_2 \mathbf{B}_l^{\mathrm{T}}) + \beta_1 ||\mathbf{W}_3||_1 + \beta_2 ||\mathbf{W}_4||_*,$$
(8)

and by setting it as $G := [I, I, D_1^T, D_2^T]^T$, the extended Lagrange function becomes

$$L(\mathbf{X}, \mathbf{W}, \mathbf{Y}, \rho) = ||\mathbf{W}_1||_* - \operatorname{Tr}(\mathbf{A}_l \mathbf{W}_2 \mathbf{B}_l^1) + \beta_1 ||\mathbf{W}_3||_1 + \beta_2 ||\mathbf{W}_4||_* + \frac{\rho}{2} ||\mathbf{G}\mathbf{X} - \mathbf{W}||_F^2 + \mathbf{Y}^{\mathrm{T}}(\mathbf{G}\mathbf{X} - \mathbf{W}).$$
(9)

We follow the update of each variable in ADMM. First, solving Equation (9) w.r.t. X yields

$$\begin{aligned} \mathbf{X}^{k+1} = & (2\mathbf{I} + \beta_1 \mathbf{D}_1^{\mathrm{T}} \mathbf{D}_1 + \beta_2 \mathbf{D}_2^{\mathrm{T}} \mathbf{D}_2)^{-1} ((\mathbf{W}_1^k - \frac{1}{\rho} \mathbf{Y}_1^k) \\ & + (\mathbf{W}_2^k - \frac{1}{\rho} \mathbf{Y}_2^k) + \beta_1 \mathbf{D}_1^{\mathrm{T}} (\mathbf{W}_3^k - \frac{1}{\rho} \mathbf{Y}_3^k) \\ & + \beta_2 \mathbf{D}_2^{\mathrm{T}} (\mathbf{W}_4^k - \frac{1}{\rho} \mathbf{Y}_4^k)), \end{aligned}$$
(10)

and applies the box constraint at the observation point to the updated \mathbf{X}^{k+1} ,

$$\mathbf{X}^{k+1} = \mathbf{X}_{\Omega^c}^{k+1} + \mathbf{M}_{\Omega}.$$
 (11)

Algorithm 2 ADMM algorithm for solving Prob. (5)
Input: $\mathbf{M}, \Omega, \mathbf{A}_l, \mathbf{B}_l, \beta_1, \beta_2, \rho$
Initialize: $X^0, W^0_1, Y^0_1, W^0_2, Y^0_2, W^0_3, Y^0_3, W^0_4, Y^0_4$
while stopping criterion is satisfied do
Update \mathbf{X}^{k+1} using (10) and (11).
$\mathbf{W}_1^{\mathrm{k}+1} = \mathcal{D}_{rac{1}{ ho}}(\mathbf{X}^{\mathrm{k}+1} + rac{1}{ ho}\mathbf{Y}_1^{\mathrm{k}})$
$\mathbf{W}_2^{\mathrm{k}+1} = \mathbf{X}^{\mathrm{k}+1} + rac{1}{ ho} (\mathbf{A}_l^{\mathrm{T}} \mathbf{B}_l + \mathbf{Y}_2^{\mathrm{k}})$
$\mathbf{W}_{3}^{k+1} = \mathcal{S}_{\frac{1}{\rho}}(\mathbf{D}_{1}\mathbf{X}^{k+1} + \frac{1}{\rho}\mathbf{Y}_{4}^{k})$
$\mathbf{W}_{4}^{\mathrm{k}+1} = \mathcal{D}_{\frac{1}{ ho}}(\mathbf{D}_{2}\mathbf{X}^{\mathrm{k}+1} + \frac{1}{ ho}\mathbf{Y}_{4}^{\mathrm{k}})$
$\mathbf{Y}_1^{k+1} = \mathbf{Y}_1^k + \rho(\mathbf{X}^{k+1} - \mathbf{W}_1^{k+1})$
$\mathbf{Y}_2^{\mathrm{k}+1} = \mathbf{Y}_2^{\mathrm{k}} + \rho(\mathbf{X}^{\mathrm{k}+1} - \mathbf{W}_2^{\mathrm{k}+1})$
$\mathbf{Y}_3^{ ext{k}+1} = \mathbf{Y}_3^{ ext{k}} + ho(\mathbf{D}_1\mathbf{X}^{ ext{k}+1} - \mathbf{W}_3^{ ext{k}+1})$
$\mathbf{Y}_4^{k+1} = \mathbf{Y}_4^k + \rho(\mathbf{D}_2\mathbf{X}^{k+1} - \mathbf{W}_4^{k+1})$
end while

Next, each auxiliary variable is updated based on the following formula,

$$\mathbf{W}_{1}^{k+1} = \arg\min_{\mathbf{W}_{1}} ||\mathbf{W}_{1}||_{*} + \frac{\rho}{2} ||\mathbf{X}^{k+1} - \mathbf{W}_{1} + \frac{1}{\rho} \mathbf{Y}_{1}^{k}||_{F}^{2}$$

$$= \mathcal{D}_{\frac{1}{\rho}}(\mathbf{X}^{k+1} + \frac{1}{\rho}\mathbf{Y}_{1}^{k}), \qquad (12)$$

$$\mathbf{W}_{2}^{k+1} = \mathbf{X}^{k+1} + \frac{1}{\rho} (\mathbf{A}_{l}^{\mathrm{T}} \mathbf{B}_{l} + \mathbf{Y}_{2}^{k}), \qquad (13)$$

$$\mathbf{W}_{3}^{k+1} = \mathcal{S}_{\frac{1}{\rho}}(\mathbf{D}_{1}\mathbf{X}^{k+1} + \frac{1}{\rho}\mathbf{Y}_{3}^{k}), \qquad (14)$$

$$\mathbf{W}_{4}^{k+1} = \mathcal{D}_{\frac{1}{\rho}}(\mathbf{D}_{2}\mathbf{X}^{k+1} + \frac{1}{\rho}\mathbf{Y}_{4}^{k}), \qquad (15)$$

where S_{τ} is a soft thresholding operator given as

$$S_{\tau}(\mathbf{X}) = \operatorname{sign}(\mathbf{X}_i) \max\{|\mathbf{X}_i| - \tau, 0\}, \quad (16)$$

and sign(·) is the sign of (·). D_{τ} is a soft thresholding operator with respect to the singular values defined by

$$\mathcal{D}_{\tau}(\mathbf{X}) = \mathbf{U}\mathcal{D}_{\tau}(\mathbf{\Sigma})\mathbf{V}^{\mathrm{T}}$$

$$(where, \mathbf{\Sigma} = \mathrm{diag}(\max\{\sigma_i - \tau, 0\}), \quad (17)$$

where

$$\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^{\mathrm{T}} \quad (where, \mathbf{\Sigma} = \mathrm{diag}(\{\sigma_i\}_{1 \le i \le r})), \qquad (18)$$

is the singular value decomposition of X.

Finally, update the dual variables based on the following formula,

$$\mathbf{Y}_{1}^{k+1} = \mathbf{Y}_{1}^{k} + \rho(\mathbf{X}^{k+1} - \mathbf{W}_{1}^{k+1}), \quad (19)$$

$$\mathbf{Y}_{2}^{\kappa+1} = \mathbf{Y}_{2}^{\kappa} + \rho(\mathbf{X}^{\kappa+1} - \mathbf{W}_{2}^{\kappa+1}), \qquad (20)$$

$$\mathbf{Y}_{3}^{\kappa+1} = \mathbf{Y}_{3}^{\kappa} + \rho(\mathbf{D}_{1}\mathbf{X}^{\kappa+1} - \mathbf{W}_{3}^{\kappa+1}), \qquad (21)$$

$$\mathbf{Y}_{4}^{k+1} = \mathbf{Y}_{4}^{k} + \rho(\mathbf{D}_{2}\mathbf{X}^{k+1} - \mathbf{W}_{4}^{k+1}).$$
(22)

These operations are summarized in Algorithm 2. Also, this algorithm ensures convergence under appropriate parameters β_1, β_2 .

III. EXPERIMENTS

In order to show the effectiveness of the proposed method, experiments were conducted on the MOCAP data restoration problem applying two kinds of deficits, random deficit and continuous deficit in the time direction, and comparison was made between the original TrNN [16] and Hu et al.'s model [17]. Missing data is randomly set by removing markers so that the missing rate is $mr = [0.2, 0.3, \ldots, 0.6]$ from each frame. Continuous missing data for the time direction is obtained by continuously removing $ml = [20, 30, \ldots, 60]$ frames with 10 markers randomly selected from each frame. As for the experimental data, samples obtained from CMU MOCAP database is used. In addition, as an evaluation index of each method in the experiment, RMSE shown in the following formula is adopted,

$$RMSE = \frac{||(\mathbf{X} - \hat{\mathbf{X}})|_{\Omega^c}||_{\mathrm{F}}}{\sqrt{|\Omega^c|}}.$$
(23)

In order to make the ranges of the evaluation values between different motion data uniform to some extent, the experiment is conducted by normalizing the whole data to a fixed mean and variance. Also, in any of the methods, the initial value was obtained by estimating the defective portion using linear interpolation.

Fig. 1 shows the results of plotting the average value of each of the three methods with respect to three different length data (Jump, Soccer, Boxing¹) acquired from CMU MOCAP Database². As for the restoration results of random missing data, there was no big difference between the conventional method [17] and the proposed method. These results show that it can be sufficiently restored by direct smoothing for instantaneous deficits such as random deficits. By contrast, for the restoration results of consecutive missing data, a critical difference was observed between the conventional method and the proposed method. As the length of the defect increases, the advantage of the proposed method becomes more prominent.

As for the destruction of acceleration/deceleration pattern due to smoothing discussed in Section I, it is conceivable that even in the proposed method, the motion pattern becomes uniform by low-rank approximation of second-order differential. Fig. 2 shows the singular values of the data obtained by taking the second derivative of the data restored by the proposed method in order to confirm the effect of regularization on the restoration results. The missing data has a larger value for the lower singular values as compared with the non-missing data, which means that the correlation of the joints with similar patterns is lost. Looking at the singular values of the restoration results by the proposed method, it is found that the distribution is close to clean data, and the lost correlation is restored. Also, since the singular value is not too small compared with the original data, it can be seen that overuniformity of the acceleration/deceleration pattern due to the

¹Motion data with indexes 13_13, 10_01, and 13_17 among the released motion data was used. ²http://mocap.cs.com.edu/

⁸⁷³



Fig. 1. RMSE of restoration results applying each method. The results of applying the random deficiency are shown in the left column, and the results of applying the continuous deficiency are shown in the right column. Results of data with different lengths are arranged, and the number of frames is 439, 801, and 4840 respectively. The longer the continuous loss length is, the more prominent the advantage of the proposed method is.



Fig. 2. A plot of singular values of a matrix obtained by the second derivative of restored data by non-missing data, missing data and proposed method. Clean data is indicated by a red solid line, missing data due to continuous missing (ml = 60) is indicated by a red dashed line, and restored data is indicated by a blue solid line. Regarding restoration data, the results of five trials are plotted. It can be understood that the correlation destroyed by the defect is repaired.

low-rank approximation of the second-order differentiation has not occurred.

From the above results, it was shown that the performance

of the proposed model is superior to that of the conventional method, for the MOCAP data restoration problem with continuous missing in the time direction.

IV. CONCLUSIONS

In this paper, we proposed a new regularization for 3D MOCAP signal restoration. It is a regularization considering the smoothness in the time direction and the correlation of each joint in the MOCAP signal, especially paying attention to the property that many of human motion accompanies smooth acceleration/deceleration. Moreover, based on the a priori knowledge that they have similar patterns among related joints, smoothing with the low-rank prior is performed. By solving the convex optimization problem using this regularization with ADMM, high precision MOCAP restoration is achieved. Experiments showed better results than the conventional method in the restoration performance of consecutive missing data.

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